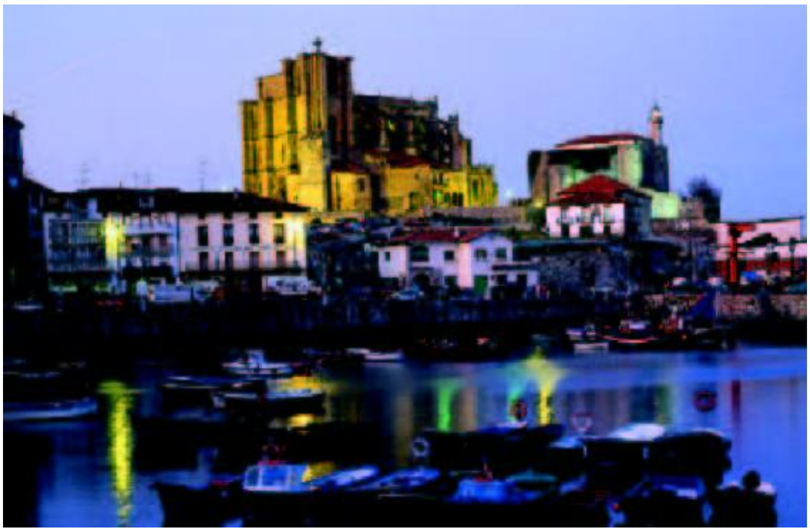
International Workshop on

The Multiscale Spectrum of Constitutive Modeling in Solid Mechanics

July 1-5 (Monday to Friday) 2019, Castro Urdiales, Spain

Organization Co-Chairmen: José Merodio, Ray Ogden



Views of Castro Urdiales, Spain

Scientific Committee

Martine Ben Amar, Ecole Normale Supérieure (France)

Davide Bigoni, University of Trento (Italy)

Roger Bustamante, Universidad de Chile (Chile)

Gal deBotton, Ben-Gurion University of the Negev (Israel)

Michel Destrade, National University of Ireland Galway (Ireland)

Luis Dorfmann, Tufts University (USA)

Marcelo Epstein, University of Calgary (Canada)

Andreas Menzel, Lund University (Sweden)

Anna Pandolfi, Politecnico di Milano (Italy)

Pedro Ponte Castañeda, University of Pennsylvania (USA)

Hosted by the Centro Internacional de Encuentros Matemáticos

Colloquium Web-site

http://www2.caminos.upm.es/Departamentos/mmcte/Personal/JMerodio/media/webconf19.pdf

Supporting Organizations

The organizers gratefully acknowledge the generous financial support from the following:

Centro Internacional de Encuentros Matematicos

Universidad de Cantabria

Ayuntamiento de Castro Urdiales

Hotel Las Rocas

Cantabria Campus Internacional

Universidad Politecnica de Madrid

University of Glasgow

ABSTRACTS

The Evolution of the Dielectric Constant in various Dielectric Elastomers subjected to Uniaxial and Biaxial Stretches

Nir Alboteanu, Gal deBotton

Department of Mechanical Engineering, Ben-Gurion University, Beer-Sheva, Israel

E-mail: niralbo@post.bgu.ac.il, debotton@bgu.ac.il

Phone number: +972 54 4882523

Keywords: Dielectrics, Electro-mechanics, Electroactive polymers.

Electroactive polymers are materials that can undergo large deformations in response to electrical stimulation. These mechanically robust, light weight and flexible materials can be used in a wide variety of applications such as robotic manipulators and vehicles, active damping, conformal control surfaces and much more [1]. The idea of using these polymers in such devices is that they essentially function as actuators with the primary part of a thin soft membrane of a DE sandwiched between two compliant electrodes. When electric potential is applied between the two electrodes the attraction between the oppositely charged electrodes cause them to be drawn to each other. This process converts electrical energy into mechanical energy.

However, despite their potential, their development is hindered by a few obstacles. The electromechanical coupling in DEs is characterized by a quadratic dependence of the force between the electrodes on the applied electric potential [2]. In turn, the deformation depends on the force via the elastic moduli. Thus, the coupling depends on the ratio between the dielectric and the elastic moduli, which is relatively low in these materials. Therefore, for a meaningful actuation high electric field is required (approximately 50 MV/m). In order to reach their potential and overcome the main obstacle, deeper understanding of their coupled electromechanical behavior is required. This, in turn, may lead to a better design of DEs that are capable of efficient operation within a safe operational envelope.

This work is aimed toward examining the coupled response of different dielectrics such as VHB and PDMS. We present an experimental apparatus which allows us to evaluate the influence of the deformation on their dielectric constant as we deepen the work performed by Cohen et al. [3]. Finally, these findings will be compared to statistical-mechanics based model [4] which takes the micro-structure of the dielectric into account.

References

[1] Frederikke B Madsen, Anders E Daugaard, Søren Hvilsted, and Anne L Skov. The current state of silicone-based dielectric elastomer transducers. Macromolecular Rapid Communications, 2016.

[2] Richard A. Toupin. The elastic dielectric. *J. Ration. Mech. Anal.*, 5:849–915, 1956.

[3] Noy Cohen, Shai Shalom Oren, and Gal deBotton. The evolution of the dielectric constant in various polymers subjected to uniaxial stretch. *Extreme Mechanics Letters*, 16:1 – 5, 2017.

[4] Noy Cohen, Kaushik Dayal, and Gal deBotton. Electroelasticity of polymer networks. *Journal of the Mechanics and Physics of Solids*, 92:105 – 126, 2016.

Tackling cancer cells plasticity in tumors: the concept of cancer stem cells

Martine Ben Amar and F. Olmeda

Laboratoire de Physique de l'Ecole normale supérieure, ENS, Université PSL, CNRS, Sorbonne Université, 24 rue Lhomond, 75005 Paris, France

Institut Universitaire de Cancérologie, Faculté de médecine, Sorbonne Université, 91 Bd de l'Hôpital, 75013 Paris, France.

Spontaneous tumor remission or relapse post treatments are oncological evidence not fully understood in growth models. A possible explanation comes from the feedback regulation of cancer stem cells via signaling molecules. According to this concept, CSCs are a small subpopulation within the tumor that are the main cause for its progression. These cells have high proliferation rate, self-renewal and immortality capacities, moreover, they are very difficult to detect and to eradicate.

The objective of our study is to deduce the morphology and fate of growing tumors under the existence of these cells exhibiting a singular behavior. A mathematical growth model is considered in the continuous limit, for a mixed population, having 3 sub-populations (CSCs), Differentiated cells (DCs) and all other cells that a tumor harbors. Analysis and stability of fixed points show the possibility of tumor extinction but also the existence of two stationary states with constant cell concentration indicating that the tumor may reach quiescent states. According to activator and inhibitor concentration which controls the phenotypic changes, a phase-diagram is presented which shows the exchange of stability as a function of the physical parameters. Focusing on the conditions for final extinction by a proper choice of the parameters is indeed a crucial goal for efficient therapeutic strategies.

The study is completed by local constraints, interaction between cells and with the environment. Nonetheless, the dynamics and patterning of tumors will be strongly affected depending on the existence of niches. Numerical investigation is then necessary varying cell motility during the tumor expansion. In particular, we show that, depending on the stroma, the cell repartition will organize differently.

WYPiWYG hyperelasticity: From macro to macro-micro-macro approaches

Victor Jesús Amores, José María Benítez, Francisco Javier Montáns
ETS de Ingeniería Aeronáutica y del Espacio. Pza Cardenal Cisneros 3, 28040 Madrid, Spain
e-mails: victorjesus.amores@upm.es, josemaria.benitez@upm.es, Fco.montans@upm.es

Abstract:

The classical approach to hyperelasticity is to assume a shape of the stored energy function, a predefined shape that is just tuned by some material parameters. These material parameters are obtained as to best-fit macroscopic data for some tests. What-You-Prescribe is What-You-Get (WYPIWYG) hyperelasticity is a different non-parametric numerical approach which does not impose the global shape of the stored energy. Instead, it is computed numerically using local interpolations and, for example, solving a linear system of equations. The initial works on WYPiWYG hyperelasticity have been phenomenological, continuum based. In these works, we have been able to obtain a stored energy function, reproducing homogeneous tests to any desired precision (hence, the name). However, as with other phenomenological models, to obtain a stored energy function which give satisfactory results for any loading condition, several tests need to be employed even in isotropic materials. Micro-macro WYPiWYG hyperelasticity [1] uses a microstructural assumption to obtain, through a pre-integration, an equivalent macroscopic expression for the stored energy function which is thereafter obtained using the previous procedures. This is an approach like that employed in the Arruda-Boyce model and, hence, a material may be characterized with a single test. However, micro-macro models require the assumption of the behavior of the constituents and their interactions and results under general loading conditions have similar accuracy than pre-integrated models like the 8-chain model.

Within the WYPiWYG framework, we propose a novel macro-micro-macro approach which considers the microstructure of the material but employs a minimum number of assumptions [2]; e.g. no assumption is made on the behavior of the constituents, on its nature, or on how interactions take pace. Instead, we interpolate the constituents' behavior and push that interpolation to the continuum level through the proper numerical integration. Then, the unknown constituents' behavior is obtained solving a linear system of equations at that continuum level, determining the microstructural behavior. From that behavior, constitutive manifolds may be generated for efficient finite element analysis of structures using that material. We show the performance of the approach predicting to excellent accuracy the series of Kawabata et al experiments employing only one of their curves to calibrate the model.

References:

[1] VJ Amores, JM Benitez, FJ Montáns (2019). Average-chain behavior of isotropic incompressible polymers obtained from macroscopic experimental data. A simple structure-based WYPiWYG model in Julia language. Advances in Engineering Software 130:41—57.

[2] VJ Amores, JM Benitez, FJ Montáns (2019). Data-driven, structure-based hyperelastic manifolds: A macro-micro-macro approach. ArXiv preprint:1903.11545, under review.

The mechanics of a “twisted” brain

Valentina Balbi v.balbiv@gmail.com
School of Mathematics, Statistics and Applied Mathematics,
NUI Galway,
Galway,
Ireland.

Biological soft tissues are particularly common in nature. For instance, many organs in the human body such as the skin, the brain, the gastro-intestinal system are made of soft tissues. The brain, among all is particularly soft and delicate. Following an impact to the skull, brain matter can experience large stretches, possibly resulting in Diffuse Axonal Injury (DAI), which is the second leading cause of death from traumatic brain injury. Previous studies have focused on linear (uni-axial) stretches of brain to investigate DAI, but in reality brain matter undergoes a mix of deformation modes during an accident. This talk will focus on the mechanical behaviour of the brain under torsion (twisting). In collaboration with University College Dublin, we collected data from torsion tests on (pigs) brain samples and modelled the experiments to finally quantify the elastic properties of the brain tissue. I will show that torsional impacts, such as a hook punch in boxing and a side impact in a car accident can also lead to dangerous levels of stretch compatible with DAI.

Essays on elastic metamaterials

Davide Bigoni

University of Trento (Trento, Italia)

In-plane wave propagation in a periodic rectangular frame structure, which includes axial and flexural deformation, the latter enhanced with rotational inertia (so-called 'Rayleigh beams'), is analyzed both with a Floquet-Bloch exact formulation for free oscillations and with a numerical treatment (developed with PML absorbing boundary conditions) for forced vibrations (including Fourier representation and energy flux evaluations), induced by a concentrated force or moment [1].

With reference to an elastic structural grid, the elements of which can sustain both axial and ?exural deformations, it is shown that material interfaces can be created with structural properties tuned by prestress states to achieve total reflection, negative refraction, and strongly localized signal channeling. The achievement of a flat lens and topologically localized modes is demonstrated, and the tunability of the system allows these properties to hold for a broad range of wavelengths [2].

Finally, invisibility for flexural vibrations in an elastic plate is addressed. A new paradigm for cloaking is introduced and its effectiveness is demonstrated both numerically and experimentally.

References

[1] Bordiga, G., Cabras, L., Bigoni, D., Piccolroaz, A. (2019) Free and forced wave propagation in a Rayleigh-beam grid: Flat bands, Dirac cones, and vibration localization vs isotropization. Int. J. Solids Structures 161, 64-81.

[2] Bordiga, G., Cabras, L., Piccolroaz, A., Bigoni, D. (2019) Prestress tuning of negative refraction and wave channeling from flexural sources. Applied Physics Letters In press.

The nonlinear response of multilayer electroactive tubes under different constraints

Eliana Bortot[1]

[1]DICAM Dpt. of Civil, Environmental and Mechanical Engineering
University of Trento, Via Mesiano 77, 38123 Trento TN Italy
E-mail: eliana.bortot@gmail.com

Dielectric elastomer transducers emerged as a promising technology for soft applications, such as actuators, generators and sensing devices [1].

Robotics applications are generally characterized by the interaction of the active membrane with a soft actuated body. Dielectric elastomer transducers produced commercially are based on non-perfectly-compliant electrodes, whose stiffness can usually not be neglected. A crucial issue in practical applications is to ensure the insulation of the active membrane so as to guarantee electrical safety of the users and electrode preservation against aggressive agents. All these three cases can be modelled by means of a three-layer system, considering the active membrane embedded between two soft passive layers [2,3].

The tubular geometry can be effectively employed in the transducer technology. Soft dielectric tubes can be actuated through the combination of a radial electric field—induced by applying a voltage drop between compliant electrodes coating its curved surfaces—and a mechanical load, for example an internal pressure [4].

The understanding of the influence of the coating layers on the active membrane response is fundamental. In the framework of nonlinear electroelasticity for heterogeneous soft dielectrics, we investigate the electromechanical response of electroactive tubes under different constraints. We consider tubes formed by the multilayer system as well as by the active membrane only.

The behaviour of the active membrane is strongly influenced by the characteristics of the coating layers, namely shear modulus and thickness. Numerical results will be presented, showing for different constraint conditions how the electromechanical response of a multilayer tube can be adjusted by modifying the properties of its constituent layers.

References

[1] Carpi, F., De Rossi, D., Kornbluh, R., Pelrine, R., and Sommer-Larsen, P. (Eds.), *Dielectric Elastomers as Electromechanical Transducers - Fundamentals, Materials, Devices, Models and Applications of an Emerging Electroactive Polymer Technology.* Elsevier, Oxford, UK, (2008).

[2] Bortot, E., "Analysis of multilayer electro-active spherical balloons" (2017) *Journal of the Mechanics and Physics of Solids*, **101**:250-267.

[3] Bortot, E., "Analysis of multilayer electro-active spherical balloons" (2019) *Journal of Intelligent Materials Systems and Structures*, **30** (1):45-62.

[4] Melnikov, A., and Ogden, R.W., "Finite deformations of an electroelastic circular cylindrical tube" (2016) *ZAMP*, **67**:140.

Statistically Isotropic neo-Hookean Composites with Spherical Inclusions

GAL DEBOTTON AND GIDON WEIL
DEPT. OF MECHANICAL ENGINEERING,
BEN-GURION UNIVERSITY,
BEER-SHEVA 8410501, ISRAEL

Abstract:
We examine the mechanical response of incompressible composites with neo-Hookean matrix and spherical neo-Hookean inclusions, within the framework of finite deformation elasticity. Following Weil and deBotton (2017, 2018) we first determine the average strain energy in a thin-wall composite sphere microstructure. On the basis of this result we derive a closed-form expression for the response of a composite sphere assemblage with a dilute concentration of the matrix phase. To the best of our knowledge this is the first time where the precise dependence of an effective SEDF of an isotropic composite on *the two isotropic invariants* I1 and I2 is explicitly determined. We further demonstrate that the dependence on I2 is small and, accordingly, bound it from above and below by neo-Hookean strain energy-density functions. Next, following the dilute iterated homogenization method of deBotton (2005), we determine the response of a composite sphere assemblage with finite volume fraction of the two phases. We end up with simple, closed-form expressions for an upper bound and a lower estimate on the effective strain energy-density function of the particulate composite. In the limit of infinitesimal deformations our solution converges to the well-known Hashin-Shtrikman bound (Hashin and Shtrikman 1963), which is slightly higher than the lower estimate, and at infinitely large deformations it agrees with a recent estimate for particulate composites Lefèvre and Lopez-Pamies (2017). Finally, we compare these findings with the few available closed-form results for finitely deforming composites with rigid inclusions. Our results are also in excellent agreement with available finite element simulations.

References:
deBotton, G. (2005). Transversely isotropic sequentially laminated composites in finite elasticity, *J. Mech. Phys. Solids* **53**: 1334–1361.
Hashin, Z. and Shtrikman, S. (1963). A variational approach to the theory of the elastic behaviour of multiphase materials, *J. Mech. Phys. Solids* **11**: 127–140.
Lefèvre, V. and Lopez-Pamies, O. (2017). Nonlinear electroelastic deformations of dielectric elastomer composites: II — non-gaussian elastic dielectrics, *J. Mech. Phys. Solids* **99**: 438–470.
Weil, G. and deBotton, G. (2017). Thin-wall composite sphere in finite shear deformation, *Int. J. Solids Struct.* **121**: 33–44.
Weil, G. and deBotton, G. (2018). Analytical and numerical analyses of neo-hookean thin-wall composite spheres, *Mathematics and Mechanics of Solids* (DOI 10.1177/1081286518807257).

Stress free 2D deformation induced by growth

Xiaoyi Chen

Beijing Normal University-Hong Kong Baptist University United International College, Zhuhai, Guangdong Province, China

Hui-Hui Dai

Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong SAR, China

Abstract:

In this talk, we consider the stress free deformation induced by growth. By imposing conditions to ensure the stress free deformation and local compatibility of deformation gradient, the governing equations (PDEs) for the deformation as well as the restrictions for the growth tensor are obtained. For a particular case, the analytical solutions for the deformation are constructed, which are applied to illustrate some interesting growth patterns. More specifically, the growth of a circular annulus and a logarithmic spiral annulus are studied. For the former one, three types of growth patterns are identified. The first is used to simulate the growth with constant thickness and a varying inner radius. The second can be used to describe the close up or open up of a circular annulus. And the third one may mimic the hollowing and anti-hollowing of some plants. On the other hand, for the growth of a logarithmic spiral annulus, the analytical solutions are obtained to describe the spread out of the spiral curves to straight lines. This growth pattern can be used to roughly describe the growth of ferns.

Modeling fiber-reinforced polymeric gels

Nikola Bosnjak[a] (nsb35@njit.edu), Howon Lee[b] (howon.lee@rutgers.edu), and Shawn A. Chester[a] (shawn.a.chester@njit.edu)

a) Mechanical Engineering
New Jersey Institute of Technology
200 Central Avenue
Newark, NJ 07102 USA

b) Mechanical and Aerospace Engineering
Rutgers University
98 Brett Road
Piscataway, NJ 08854 USA

Polymeric materials consist of long chain of molecules, which form a polymer network. When a polymer network is exposed to a suitable solvent, the solvent molecules are able to diffuse inside the network, causing it to undergo a volumetric deformation, known as swelling. A polymeric material in this mixed and swollen state is known as a polymeric gel. Polymeric gels are ubiquitous, they are found in a wide variety applications including: soft robots; packers in the oil industry; valves in microfluidic channels; and a diverse array of biomedical applications. Many soft robots and biological tissues are gel-like in constitution and structure, which makes a thorough mechanistic understanding of these materials essential for our understanding their mechanical response. An important distinction between these applications and much of the previous mechanics literature on gels is that most soft robotics and biological tissues contain embedded fibers. The existence of these embedded fibers imposes pronounced anisotropy in the response of these materials, and plays an important role in operation.

The objective of this research is to develop a continuum level coupled diffusion-deformation constitutive model for fiber-reinforced polymeric gels. The model builds upon previous work by taking into account the mechanical influence of fibers, as well as anisotropic diffusion. For modeling the mechanical behavior of the polymer matrix, we adopt a non-Gaussian statistical mechanics based model, which takes into account limited extensibility of polymer chains. The influence of multiple families of embedded fibers on the mechanical response of the polymeric gel is modeled using a fiber volume fraction, modulus, and fiber direction, for each family. To account for anisotropy in the fluid diffusion, we consider a tensorial diffusivity. The constitutive model is numerically implemented in the software package Abaqus by writing a user element subroutine (UEL). We showcase the capabilities of our model to simulate the major aspects of fiber-reinforced gel behavior, including anisotropic diffusion and the influence of different fiber volume fractions and orientations. Lastly, the constitutive model is applied to simulate the operation of a diffusion activated soft gripper, and compared to a manufactured a soft gripper, containing embedded fibers, which is used to pick up an object.

Drastic swelling-induced softening of hydrogen-bond dominated biopolymer networks

Noy Cohen
Department of Materials Science and Engineering, Technion - Israel Institute of Technology, Israel
noyco@technion.ac.il

Claus D. Eisenbach
Materials Research Laboratory, University of California, Santa Barbara, CA, USA
Institute for Polymer Chemistry, University of Stuttgart, Germany

The penetration of water into rubber-like protein networks such as cross-linked resilin, which is found in insects, can lead to changes in stiffness that range over several order of magnitudes. This softening effect cannot be explained by the volumetric changes associated with pure swelling/deswelling that is used to describe standard polymer networks such as rubber. Rather, this property stems from the reversible swelling-induced breaking of hydrogen cross-linking bonds that connect the chains in the network. In this talk, we present a microscopically motivated model for the swelling and the mechanical response of hydrogen-bond dominated biopolymer networks. It is shown that the penetration of water molecules into the network leads to the breaking of non-covalent crosslinking sites. In turn, the network experiences a reduction in the effective chain-density and an increase in entropy (leading to a decrease in free energy), thus explaining the dramatic softening. Additionally, the breaking of bonds alters the micro-structure and changes the quantitative elastic behavior of the network. The proposed model is found to be in excellent agreement with several experimental findings. The merit of this work is two-fold: (1) it accounts for the number and the strength of non-covalent cross-linking bonds, thus explaining the drastic reduction in stiffness upon water uptake, and (2) it provides a method to characterize the microstructural evolution of hydrogen-bond dominated networks. Consequently, the model can be used as a micro-structural design-guide to program the response of synthetic polymers.

Wrinkling in voltage and charge-controlled soft dielectric plates

Hannah Conroy Broderick
hannah.conroybroderick@nuigalway.ie
School of Mathematics, Statistics and Applied Mathematics,
NUI Galway,
Galway,
Ireland.

Dielectric materials are smart materials that deform elastically in the presence of an electric field. They have potential applications in devices such as artificial muscles and soft robotics, where there is demand for materials that can undergo repeated large deformations.

We show that a smooth giant voltage actuation of soft dielectric plates is not easily obtained in practice. In principle one can exploit, through pre-deformation, the snap-through behaviour of the voltage-control loading curve to deliver a large stretch prior to electric breakdown. However, we demonstrate that even in this favourable scenario, the soft dielectric is likely to first encounter the plate wrinkling phenomenon, as modelled by the onset of small-amplitude sinusoidal perturbations on its faces.

We also investigate the case of a soft dielectric plate deformed by the coupled effects of a mechanical pre-stress applied on its lateral faces and an electric field applied through its thickness under charge-controlled actuation, where the electric field is created by spraying charges on the major faces of the plate. Although in practice this mode of actuation is harder to achieve than a voltage-driven deformation, here we find that it turns out to be much more stable in theory. We show that the geometric instability associated with the formation of small-amplitude wrinkles on the faces of the plate that arises under voltage control does not occur in this case. Further, using Finite Element simulations, we find that the actuation is limited by a breakdown due to inhomogeneous fields developing close to the clamps of the plate.

This is joint work with Michel Destrade, Yipin Su, Weiqiu Chen, Michele Righi and Ray W. Ogden.

On the growth-induced 2D shape programing through a finite-strain plate theory

Hui-Hui Dai

Department of Mathematics

City University of Hong Kong

Email: mahhdai@cityu.edu.hk

Growth-induced plane-strain deformations of hyperelastic plates are studied. First, we brief recall a finite-strain plate theory with growth field incorporated, which is directly derived from the 3D differential formulation.

Then, specializing to a plane-strain set, we obtain two coupled nonlinear ODEs. By proposing a novel transformation, these two equations are solved analytically and the solutions are explicitly represented in terms of growth functions.

The inverse problem can also be solved, leading to analytical formulas for the growth functions in terms of the two unknown position components. We then use them to identify the growth functions for generating arbitrary 2D

geometrical shapes of the hyperelastic plates. A few examples are present, including growth-induced circular, butter-fly, elliptic and helical shapes, which are verified by numerical simulations. In particular, we show how a plate can

grow into an Euler's spiral.   Our analytical results can also capture the bending deformation of a crosslinked PDMS beam due to swelling, which was reported in an experiment.

References:

1. J. Wang, D. Steigmann, F. Wang and H.-H. Dai, On a consistent finite-strain plate theory of growth, J. Mech. Phys. Solids, 2018, 111, 184–214.
2. J. Wang, Q. Wang, H.-H. Dai,   P. Du and D. Chen, Shape-programming of hyperelastic plates through differential growth: an analytical approach, Soft Matter, 2019, 15, 2391

The role of porosity and solid matrix compressibility on the mechanical behavior of poroelastic tissues

Hamidreza Dehghani[1], Raimondo Penta[2], José Merodio[1]

[1] Continuum Mechanics Department, Universidad Politecnica de Madrid, Madrid, Spain. E-mail contacts: dehghanihamid2@gmail.com; merodioj@gmail.com

[2] School of Mathematics and Statistics, University of Glasgow, UK

E-mail contact: Raimondo.Penta@glasgow.ac.uk

ABSTRACT

We investigate the dependence of the mechanical and hydraulic properties of poroelastic materials on the interstitial volume fraction (*porosity*) of fluid flowing through their pores and compressibility of their elastic (*matrix*) phase.

The mechanical behavior of the matrix is assumed of linear elastic type and we conduct a three-dimensional microstructural analysis by means of the asymptotic homogenization technique exploiting the length scale separation between the pores (pore-scale or *microscale*) and the average tissue size (*macroscale*).

The coefficients of the model are therefore obtained by suitable averages which involve the solutions of periodic cell problems at the pore-scale [I]. The latter are solved numerically by finite elements in a cubic cell by assuming a cross-shaped interconnected cylindrical structure which results in a cubic symmetric stiffness tensor on the macroscale. Therefore, the macroscale response of the material is fully characterized by six parameters, namely the elastic Young's and shear moduli, Poisson's ratio, the hydraulic conductivity, and the poroelastic parameters, i.e. Biot's modulus and Biot's coefficient.

We present our findings in terms of a parametric analysis conducted by varying the porosity as well as the Poisson's ratio of the matrix. Our novel three-dimensional results [II], which are presented in the context of tumor modeling, serve as a robust first step to (a) quantify the macroscale response of poroelastic materials on the basis of their underlying microstructure, (b) relate the compressibility of the tissue, which can be used to distinguish between benign tumor and cancer, to its microstructural properties (such as porosity), and (c) reveal a nontrivial dependency of Biot's modulus on porosity and compressibility of the matrix, which can pave the way to the optimal design of artificial constructs in terms of fluid volume available for transport of mass and solutes.

REFERENCES

[I] Burridge, Robert, and Joseph B. Keller. "Poroelasticity equations derived from microstructure." The Journal of the Acoustical Society of America 70.4 (1981): 1140-1146.

[II] Dehghani, H., R. Penta, and J. Merodio. "The role of porosity and solid matrix compressibility on the mechanical behavior of poroelastic tissues." *Materials Research Express* 6.3 (2018): 035404.

Bending Bifurcation and post-bifurcation of an inflated and extended residually-stressed circular cylindrical tube with application to abdominal aortic aneurysms

Dariel Desena, Carlos de los Santos, Hamidreza Dehghani and Jose Merodio

Department of Continuum Mechanics and Structures, Technical University of Madrid, Madrid, Spain

A numerical procedure to analyze bifurcation and post-bifurcation of a finite deformation boundary-value problem for a residually-stressed elastic body is studied. In particular, the problem is the combined extension and inflation of a circular cylindrical tube subject to radial and circumferential residual stresses. The material model, given by a residual-stress dependent nonlinear elastic constitutive law in terms of invariants, is implemented in a finite element code. A numerical procedure to analyze the bifurcation and post-bifurcation of the finite deformation boundary-value problem at hand is developed based on the modified Riks method.The dependence of bifurcation and post-bifurcation behavior of tubes under the loading at hand on residual stresses is shown and compared with results when there is no residual stress. The finite deformation boundary-value problem is described mainly in terms of the inflation pressure, as well as the axial and azimuthal stretches of the tube. The dependence of these quantities on bifurcation is illustrated graphically for different values of the parameters (in dimensionless form) involved, in particular, the strength of the residual stress. It is found that for small values of the axial stretch (close to the non-extended configuration), the onset of bifurcation is found to be the bending mode. Furthermore, for the latter case in subsequent motion, i.e. post-bifurcation, it is shown that bending triggers bulging as opposed to the situation in which bending is not allowed and the onset of bifurcation is associated with bulging. In addition, the bulge, or the abnormal enlargement, that is formed during post-bifurcation after the onset of bending bifurcation appears on one side of the tube showing an irregular shape which is consistent with the development of abdominal aortic aneurysms (AAA).

Propagation of Weakly Nonlinear Waves in Nearly Incompressible Isotropic Elastic Solids

Włodzimierz Domański[1], *Stanisław Jemioło* [2], *Aleksander Franus*[2]

[1]Military University of Technology, Faculty of Cybernetics,
Institute of Mathematics and Cryptology, Urbanowicza 2, 00-908 Warsaw 49, Poland
domanski.wlodek@gmail.com

[2]Warsaw University of Technology, Faculty of Civil Engineering,
Institute of Building Engineering, al. Armii Ludowej 16, 00-637 Warsaw, Poland
s.jemiolo@il.pw.edu.pl a.franus@il.pw.edu.pl

We analyse propagation of weakly nonlinear elastic plane waves in rubberlike materials. We consider the class of nearly incompressible isotropic hyper-elastic constitutive relationships of such materials. We assume that the stored energy function consists of two parts: deviatoric and volumetric:

$$W\left(\bar{I}_1,\bar{I}_2,J\right)=W_D\left(\bar{I}_1,\bar{I}_2\right)+W_V\left(J\right) , \tag{1}$$

where $\bar{I}_1,\bar{I}_2$ are principal invariants of the isochoric right deformation tensor $\bar{\mathbf{C}}=\bar{\mathbf{F}}^T\bar{\mathbf{F}}$ with $\bar{\mathbf{F}}=J^{-1/3}\mathbf{F}$ and $J=\det\mathbf{F}$, where $\mathbf{F}$ is the gradient of deformation. We analyse different forms of functions $W_D\left(\bar{I}_1,\bar{I}_2\right)$ and $W_V\left(J\right)$. Next, we approximate them with respect to the principal invariants of Green strain tensor $\mathbf{E}$. We express the energy function (1) in terms of invariants of $\mathbf{E}$ up to the third order. In this way we obtain the stored energy function in the form of the compressible Murnaghan material. However, in our case we get less parameters which characterise rubberlike materials.

We derive closed formulas for the velocities of plane waves propagating in materials characterised by the stored energy function (1). We can express them with the help of material parameters of function (1). In the general case they can be written in terms of the derivatives of the energy functions $W_D\left(\bar{I}_1,\bar{I}_2\right)$ and $W_V\left(J\right)$.

Finally, using the method of weakly nonlinear asymptotics we derive evolution equations for the amplitudes of longitudinal and shear elastic waves propagating in considered rubberlike materials.

Mechanics, Modeling and Simulation of Aortic Dissection: The LEAD Project of TU Graz

Gerhard A. Holzapfel (a) (b), Malte Rolf-Pissarczyk(a), Kewei Li(a)

(a)Graz University of Technology, Institute of Biomechanics, Austria
e-mail: m.rolf-pissarczyk@tugraz.at, kewei.lig@tugraz.at

(b)Norwegian University of Science and Technology (NTNU)
Department of Structural Engineering, Norway
e-mail: gerhard.holzapfel@ntnu.no

In the LEAD project a consortium of scientists from biomechanical-, civil-, electrical-, and mechanical engineering, computer science, mathematics, and physics from TU Graz has set itself the goal of unraveling the cause and the formation of the various stages of an aortic dissection (AD) [1]. Advanced computational tools and algorithms will be developed to assist clinicians with the diagnosis, treatment, and management of AD patients. In addition, related topics such as the optimization of implants, the better design for tissue engineering and of coatings and stent platforms for drug delivery will be investigated.

One aspect is the development of new multiscale constitutive models that include parameters and failure criteria, which will allow the simulation of the rupture of aortic tissues and the propagation of the false lumen. It is suggested that the propagation of the dissection is often induced by the degradation of elastin in the media [2]. By studying the degradation of elastic fibers in the aortic wall, a possible role is giving to the accumulation of pooled glycosaminoglycans [2]. They can induce a swelling pressure between the elastic lamellae in the extracellular matrix causing damage of interlamellar elastic fibers which interconnect either with smooth muscle cells or the elastic lamellae. Apart from providing a brief overview of the LEAD project we propose a constitutive model to describe the degradation of elastic fibers. In particular, the discrete fiber dispersion method is used to incorporate the contributions of
dispersed collagen and elastic fibers in the aortic wall [3]. Representative numerical examples are chosen to demonstrate the performance of the proposed constitutive model with a locally defined degradation parameter.

We expect that the LEAD project will improve awareness of this life-threatening disease, and lead to a more effective treatment and control within the general public of Austria and beyond.

REFERENCES

[1] LEAD Project on ‘Mechanics, Modeling and Simulation of Aortic Dissection’, granted by Graz University of Technology, Austria: www.biomechaorta.tugraz.at.
[2] J. D. Humphrey. Possible mechanical roles of glycosaminoglycans in thoracic aortic dissection and associations with dysregulated transforming growth factor-_. J. Vasc. Res., 50:1–10, 2013.
[3] K. Li, R. W. Ogden, and G. A. Holzapfel. A discrete fibre dispersion method for excluding fibres under compression in the modelling of fibrous tissues. J. R. Soc. Interface, 15:20170766, 2018.

The coupling of bioelectrical and mechanical phenomena

Alessandro Leronni, Lorenzo Bardella

Department of Civil, Environmental, Architectural Engineering, and Mathematics

University of Brescia, Brescia, Italy

Alexis Pietak

Allen Discovery Center, Tufts University, Medford, MA 02155, USA

Luis Dorfmann

Department of Civil and Environmental Engineering, Tufts University, Medford, MA 02155, USA

Pietak and Levin [1] investigate the role of bioelectricity in pattern regulation by developing a finite volume-based multiphysics simulator referred to as the BioElectric Tissue Simulation Engine (BETSE). In particular, the dynamic of bioelectricity is described in terms of the spatiotemporal evolution of the electric potential, and the relevant ion concentrations over a network of cells interacting with each other and with their environment through their plasma membrane. In this study, the BETSE platform is augmented to account for the mechanical interactions between cells, which arise from bioelectrical ion fluxes. On the basis of the electrochemomechanical theories developed in the last decade [2, 3, 4, 5], the electrostatic force and osmotic pressure, respectively generated by voltage and concentration gradients across the cell membranes, are used to determine the mechanical stress. Then, assuming an isotropic response, the deformation field of the cell cluster is obtained. The whole electrochemomechanical process is in principle coupled, as the deformation may trigger the opening of mechano-sensitive ion channels, ultimately leading to a reshaping of the spatial ion fluxes and membrane potential patterns within the cell network.

References:

Pietak A, Levin M 2016 Exploring instructive physiological signaling with the bioelectric tissue simulation engine. Front. Bioeng. Biotechnol. **4**, 55

Hong W, Zhao, Suo Z 2010 Large deformation and electrochemistry of polyelectrolyte gels. J. Mech. Phys. Solids. **58**, 558-577

Cha Y, Porfiri M 2014 Mechanics and electrochemistry of ionic polymer metal composites. J. Mech. Phys. Solids. **71**, 156-178

Dorfmann L, Ogden RW 2017 Nonlinear electroelasticity: material properties, continuum theory and applications, Proc. R. Soc. A **473**, 20170311

Leronni A, Bardella L 2019 Influence of shear on sensing of ionic polymer metal composites. Eur. J. Mech. A-Solid. In press

Influence of Initial Residual Stress on Growth and Pattern Creation for a Layered Aorta

Yangkun Du, Chaofeng Lu, Michel Destrade, and Weiqiu Chen

Abstract: Residual stress is ubiquitous and indispensable in most biological and artificial materials, where it sustains and optimizes many biological and functional mechanisms. The theory of volume growth is widely used to explain the creation and evolution of growth-induced residual stress and the resulting changes in shape, starting from a stress-free initial state. Furthermore, the theory can also model how growing bio-tissues such as arteries and solid tumors develop a strategy of pattern creation according to geometrical and material parameters, again starting from a stress-free initial stress. This modelling provides promising avenues for designing and directing some appropriate morphology of a given tissue or organ and achieve some targeted biomedical function. In this paper, we rely on a modified, augmented theory to reveal how we can obtain growth- induced residual stress and pattern evolution of a layered artery by starting from an existing, non-zero initial residual stress state. We use experimentally determined residual stress distributions of aged bi-layered human aortas and quantify their influence by a magnitude factor. Our results show that initial residual stress has a most significant impact on residual stress accumulation and the subsequent evolution of patterns. Additionally, we provide an essential explanation for growth-induced patterns driven by differential growth coupled to an initial residual stress. Finally, we show that initial residual stress is a readily available way to control growth-induced pattern creation for tissues and thus, a promising inspiration for biomedical engineering.

A Model for the Mechanical feedback Control of the Transformation of Fibroblasts to Myofibroblasts*

Elliot L. Elson [a, *], Hong Qian [b], Judy A. Fee [a], Tetsuro Wakatsuki [c]

The phenotypic transformation of normal fibroblasts to myofibroblasts is central to normal wound healing and to pathological fibrosis that can occur in the heart and many other tissues. The transformation occurs in two stages. The first stage is driven mainly by mechanical changes such as increased stiffness of the heart due to hypertension and cellular contractility. The second stage requires both increasing stiffness and biochemical factors such as the growth factor, TGFbeta. As more and more cells convert from weakly contractile fibroblasts to strongly contractile myofibroblasts, the stiffness of the ventricular muscle increases. This increased stiffness can impair cardiac function. We illustrate the large mechanical differences between normal fibroblasts and myofibroblasts with measurements in engineered tissue constructs. We also propose a simple model for the establishment of non-equilibrim steady states with different compositions of fibroblasts and myofibroblasts. Under some conditions a positive feedback loop resulting from the increasing stiffness caused by increasing numbers of myofibroblasts can produce a bifurcation between steady states with low and high myofibroblast content.

*Elson, E. L., H. Qian, J. A. Fee and T. Wakatsuki (2018). "A model for positive feedback control of the transformation of fibroblasts to myofibroblasts." Prog Biophys Mol Biol.

[a] Department of Biochemistry and Molecular Biophysics, Washington University in St. Louis, School of Medicine, Campus Box 8231, 660 S. Euclid Avenue, St. Louis, MO, 63110, USA

[b] Department of Applied Mathematics, University of Washington, Lewis Hall 201 Box 353925, Seattle, WA, 98195, USA

[c] InvivoSciences, Inc, 510 Charmany, Madison, WI, 53719, USA

GRASSMANN STATICS

By

Marcelo Epstein

University of Calgary, Canada

Abstract: The conventional notion of mechanical interactions as force-vectors in the standard three-dimensional Euclidean space E^3 is extended to include multi-vectors of all orders in an affine space of arbitrary dimension. Within this general framework, the concepts of screws and counter-screws can be rigorously defined, including a non-metric extension of the principle of virtual work without invoking a dual space. Even in the case of E^3, applications can be expected in the realm of media with internal structure. A historically interesting illustration of the theory is obtained in terms of Vlasov's theory of thin-walled beams of open cross section.

Comparison of Parameters Identification Methods for Hyperelastic Models of Rubberlike Materials

Aleksander Franus[1]*, Stanisław Jemioło*[1]*, Włodzimierz Domański*[2]

[1]Warsaw University of Technology, Faculty of Civil Engineering,
Institute of Building Engineering,
al. Armii Ludowej 16, 00-637 Warsaw, Poland

a.franus@il.pw.edu.pl, s.jemiolo@il.pw.edu.pl

[2]Military University of Technology, Faculty of Cybernetics,
Institute of Mathematics and Cryptology, Urbanowicza 2, 00-908 Warsaw 49, Poland

domanski.wlodek@gmail.com

The paper addresses a comparison of parameter identification methods for hyperelastic models of rubberlike materials. We consider isotropic incompressible materials, assuming that $J = \det \mathbf{F} = 1$, where $\mathbf{F}$ stands for the deformation gradient. However, we define the strain energy function on the basis of only first $\bar{I}_1 = \mathrm{tr}\bar{\mathbf{C}} = \mathrm{tr}\bar{\mathbf{B}}$ and second $\bar{I}_2 = \mathrm{trCof}\bar{\mathbf{C}} = \mathrm{trCof}\bar{\mathbf{B}}$ invariants of the isochoric right and left Cauchy-Green tensors:

$$W = W_D(\bar{I}_1, \bar{I}_2)\,. \qquad (1)$$

with $\bar{\mathbf{C}} = \bar{\mathbf{F}}^T\bar{\mathbf{F}}$ and $\bar{\mathbf{F}} = J^{-1/3}\mathbf{F}$.

Several specific functions of the type (1) are considered. The main distinction concerns the fact of linearity or nonlinearity of these functions with respect to material parameters. Polyconvexity requirements are taken into account as well, which are reflected in certain constrains of the parameters.

We consider three approaches to the parameter identification, that is, the approximation is carried out based on the values of the Cauchy stress, the nominal stress and the corresponding stored energy function, respectively. Mathematically, the corresponding approximation task is stated as an optimisation problem with constrains. Moreover, we investigate the minimization of absolute and relative errors in the formulation of the problem.

Parameters of certain models of the form (1) are identified based on the values obtained in uniaxial and biaxial tension experiments. The approach is adequate, because of the incompressibility assumption of the materials. The values are approximated simultaneously in every method. Special attention is paid to constrains on values of parameters in the context of physical interpretation, so that we can distinguish characteristic phases in typical behaviour of rubberlike materials in tension tests.

Elastic waves in two-phase composite quasicrystalline-generated metamaterials: scaling of frequency spectra and negative refraction

Massimiliano Gei[1], Lorenzo Morini[1]

[1]*School of Engineering, Cardiff University, The Parade, Cardiff CF24 3AA, UK*
E-mail: *geim@cardiff.ac.uk*

Keywords: metamaterial, phononic crystal, Fibonacci structure.

The talk refers to a detailed investigation of scaling and self-similarity of the frequency spectra of axial waveguides composed of repeated elementary cells designed by adopting the class of quasicrystalline Fibonacci substitution rules. For this problem, an invariant function of the circular frequency, the Kohmoto's invariant, governs self-similarity and scaling of the stop/pass band layout within defined ranges of frequencies at increasing generation index [1,2]. The Kohmoto's invariant also explains the existence of specific frequencies, named *canonical frequencies*, associated with closed orbits on the geometrical three-dimensional representation of the invariant.
In the second part, the problem of an antiplane wave obliquely incident at the interface between an elastic substrate and a quasicrystalline laminate is investigated. The substrate-laminate system is studied by combining the transfer matrix method to the normal mode decomposition. The refraction angles associated with the transmitted modes are estimated by means of the space averaging procedure of the Poynting vector [3]. We show that the Floquet-Bloch spectrum corresponding to this class of laminates is characterized by a self-similar structure similar to that observed for axial waveguides studied previously. Moreover, high-order Fibonacci laminates can provide pure negative wave refraction at lower frequencies with respect to the periodic two-phase multilayered materials used for the design of several phononic devices [3].
The obtained results represent an important advancement towards the realisation of composite quasicrystalline metamaterials.

References

[1] Gei, M. (2010) Wave propagation in quasiperiodic structures: stop/pass band distribution and prestress effects, International Journal of Solids and Structures 47, 3067-3075.
[2] Morini, L., Gei, M. (2018) Waves in one-dimensional quasicrystalline structures: dynamical trace mapping, scaling and self-similarity of the spectrum, Journal of the Mechanics and Physics of Solids 119, 83-103.
[3] Morini, L., Eyzat, Y. and Gei, M. (2019) Negative refraction in quasicrystalline multilayered metamaterials. Journal of the Mechanics and Physics of Solids 124, 282-298.

Stress projections and data-driven modelling

Giulio G. Giusteri[1], Raimondo Penta[2], Ryohei Seto[3]

[1]*Dipartimento di Matematica, Università degli Studi di Padova, via Trieste 63, 35121, Padova, Italy. E-mail:* giulio.giusteri@math.unipd.it
[2]*School of Mathematics and Statistics, University of Glasgow, University Place, Glasgow G12 8QQ, United Kingdom. E-mail:* raimondo.penta@glasgow.ac.uk
[3]*Department of Chemical Engineering, Kyoto University, Kyoto 615-8510, Japan. E-mail:* setoryohei@me.com

Abstract:
Representation theorems and integrity bases for tensorial functions are basic tools in the construction of continuum mechanical models. In spite of their importance for constitutive theories, these concepts may be difficult to apply in the context of data-driven modelling of composite or micro-structured materials for two reasons. First, representation theorems are based upon a priori assumptions about the list of independent descriptors of the continuum (in terms of which the stress is represented), but, when dealing with complex materials, it may be too restrictive to make such assumptions, due to our limited knowledge of phenomena at the micro-scale. Second, it is experimentally challenging (if at all possible) to independently excite the different terms of an integrity basis and obtain from the data a meaningful fitting of the several material coefficients present in such representations of the stress tensor.
To improve our capability of assimilating experimental and computational data obtained in different conditions and building continuum mechanical models directly based on the collected data, we propose the use of suitably defined orthogonal projections of the stress tensor. These lead to linear decompositions of the stress on orthonormal tensorial bases that can be adapted to various local characteristics of a material deformation. Such decompositions are universal in the sense that the tensorial bases are independent of any constitutive prescription about the material behaviour. Moreover, the orthogonality properties allow to link the different coefficients of a decomposition to specific effects that can usually be measured independently. For all these reasons, similar stress projections can play an important role in the data-driven modelling of multi-scale materials and in the broader context of nonlinear elasticity.

Growing media and "internal time"

Alfio Grillo[1], Salvatore Di Stefano[1], Salvatore Federico[2]

[1]Dipartimento di Scienze Matematiche (DISMA) "G.L. Lagrange", Politecnico di Torino, *"Dipartimento di Eccellenza 2018-2022"*, C.so Duca degli Abruzzi 24, 10129, Torino, Italy

[2]Department of Mechanical and Manufacturing Engineering, The University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N1N4, Canada

E-mail: alfio.grillo@polito.it, salvatore.distefano@polito.it, salvatore.federico@ucalgary.ca

Abstract

In this contribution, which summarizes some results outlined in [1], we take inspiration from the notion of *"thermodynamic time"* [2] to introduce a scalar field that defines an indicator of the material inhomogeneities [3] induced by growth in a continuum medium. Such scalar field is identified with the solution of a partial differential equation, obtained by exploiting the medium's energy balance, and is meant to individuate the time scale with which the growing medium changes its internal structure. For this reason, we call it *"internal time"* [1]. Given a medium, in which growth is described by means of a growth tensor, we determine its internal time in two ways, based on different conceptions of growth: the growth tensor is viewed as an internal variable in the first case [3], and as a kinematic descriptor in the second one [4]. Then, as a proof of concept, we compute the internal time for a benchmark problem in which growth is modelled according to the two philosophies mentioned above.

References

[1] Grillo, A., Di Stefano, S., Federico, S.: "Growth and remodelling from the perspective of Noether's Theorem". *In preparation.*

[2] Mićunović, M.M.: *"Thermomechanics of Viscoplasticity"*, Springer New York, 2009.

[3] Epstein, M., Maugin, G.A.: "Thermomechanics of volumetric growth in uniform bodies". *International Journal of Plasticity*, **16** (2000) 951-978.

[4] DiCarlo, A., Quiligotti, S.: "Growth and balance", *Mechanics Research Communications*, **29** (2002) 449-456.

ON THE AIC BASED MODEL REDUCTION FOR THE GENERAL HOLZAPFEL-OGDEN MYOCARDIAL CONSTITUTIVE LAW

Debao Guan (1), Faizan Ahmad (2), Peter Theobald (2), Shwe Soe (2), Xiaoyu Luo* (1), Hao Gao (1)

1. School of Mathematics and Statistics, University of Glasgow, UK;
2. School of Engineering, Cardiff University, UK
*Corresponding author: Xiaoyu.luo@glasgow.ac.u

Constitutive laws that describe the mechanical responses of cardiac tissue under loading hold the key to accurately modelling the biomechanical behaviour of the heart. There have been ample choices of phenomenological constitutive laws derived from experiments, some of which are quite sophisticated and include effects of microscopic fibre structures of the myocardium. A typical example is the 8-parameter strain-invariant based Holzapfel-Ogden (HO) 2009 model [1] that is excellently fitted to simple shear tests [4]. It has been widely used and regarded as the state-of-the-art constitutive law for myocardium. However, there has been no analysis to show if it has both adequate descriptive and predictive capabilities for other tissue tests of myocardium. Indeed, such an analysis is important for any constitutive laws for clinically useful computational simulations. In this work, we perform such an analysis.
We analysis the HO constitutive laws using combinations of tissue tests, uniaxial tension, biaxial tension, and simple shear from three different sets of myocardial tissue studies [2-4]. We start from the general 14-parameter myocardial constitutive law developed by Holzapfel and Ogden [1]. We show that this model has good descriptive and predictive capabilities for all the experimental tests. However, to reliably determine all 14 parameters of the model from experiments remains a great challenge. Hence, we derive three different reduced forms of the constitutive law using the Akaike information criterion (AIC) [5], aiming to maintain the mechanical integrity whilst achieving minimal computational cost. One of the reduced forms turns out to be the widely used 8-parameter HO model for myocardium [1]. To improve the fittings, we exclude the upstretched fibres using an “effective fibre ratio”, which depends on the sample size, shape, local myofibre architecture, and loading conditions. Finally, we also use AIC method to study the optimal combinations of tissue tests for a given constitutive model.

Our results show that different reduced forms of the HO model can be derived depending on what experimental data are used, since not all properties of the myocardium may be measured from one set of tests. Moreover, for a given model, we can identify a minimum set of experiments required to determine the material properties. For example, we found that one may use just one shear responses (along normal-fibre direction) and one biaxial stretch (ratio of 1 mean fibre: 1 cross-fibre) from [4] to satisfactorily describe human myocardial mechanical properties. Our study also shows that single-state tests (i.e. simple shear or stretching only) are insufficient to determine the myocardium responses. It is also important to consider the transmural fibre rotation within each myocardial sample of the tests.

References

1. Holzapfel GA, Ogden RW (2009) Constitutive modelling of passive myocardium: a structurally based framework for material characterization. Phil. Trans. of RS A: Mathematical, Physical and Engineering Sciences 367(1902):3445–3475.
2. Dokos S, Smaill BH, Young AA, LeGrice IJ (2002) Shear properties of passive, ventricular myocardium. American J. of Physio-Heart and Circ Physio 283(6):H2650–H2659.
3. Ahmad F, Liao J, Soe S, Jones MD, Miller J, Berthelson P, Enge D, Copeland KM, Shaabeth S, Johnston R, et al (2018). Biomechanical properties and microstructure of neonatal porcine ventricles. J MBBM 88:18–28.
4. Sommer G, Schriefl AJ, Andr¨a M, Sacherer M, Viertler C, Wolinski H, Holzapfel GA (2015) Biomechanical properties and microstructure of human ventricular myocardium. Acta Biomaterialia 24:172–192.
5. Schmid H, Nash MP, Young AA, Hunter PJ (2006) Myocardial material parameter estimation-a comparative study for simple shear. J. of BioMech. Eng. 128(5):742

Acknowledgements

We are grateful for funding provided by UK EPSRC (EP/N014642/1). D. Guan also acknowledges funding from Chinese Scholarship Council and the fee waiver from the University of Glasgow. F. Ahmad is grateful to the Ser Cymru NRN in Advanced Engineering & Materials for funding his PhD scholarship.

DISCRETE-TO-CONTINUUM MODELLING OF CELLS TO TISSUE

Nicholas A. Hill, Roxanna G. Barry, Peter S. Stewart

Nicholas.Hill@glasgow.ac.uk, *r.barry.1@research.gla.ac.uk*, *Peter.Stewart@glasgow.ac.uk*

SofTMech & School of Mathematics & Statistics, University of Glasgow, U.K.

Living tissues are composed of large numbers of cells packed together within an extracellular matrix. In order to understand the process of growth and remodelling in soft tissues that are subject to internal and external forces and strains, multiscale models that describe the interactions between individual cells and the tissue as a whole are needed. A significant challenge in multiscale modelling of tissues is to produce macroscale continuum models which rationally encode behaviour from the microscale (discrete cells). Over the years a number of highly successful approaches have been developed to rationally form macroscale models for multiscale processes such as solute transport and cell-cell signalling. However, such approaches have focused on homogenization techniques, which typically rely on underlying symmetries or periodicity on the smaller scales. We address the need for models that rationally incorporate the underlying mechanical properties of individual cells, without assuming homogeneity, symmetry or periodicity at the cell level. This challenge is particularly pertinent in modelling cardiac tissue, where the individual cells experience significant mechanical deformation in response to (periodic) electrical signalling. In particular, we are interested in cases where the mechanical properties of the cardiac cells may vary significantly between different regions of the heart (e.g. in disease or following a myocardial infarction).

We consider a single line of nonlinearly hyperelastic cells of finite size, with forces transmitted across the boundaries between neighbours. One or both ends of the line are fixed to represent free expansion or confinement. The dynamics of the array is given by a system of discrete 1D ODE's. Individual cells grow in volume and divide into two identical daughter cells. The parent cell divides its mass equally so that each daughter cell is half the total length of the parent cell, and an extra boundary at the midpoint of the parent cell is introduced. Two examples of resistance to motion are considered.

Firstly, we suppose that the cells are binding and unbinding to a fixed substrate, providing a resistive force is proportional to the speed of the cell relative to the substrate (Stokes dissipation). Secondly, we consider a local resistance to motion arising from the motion of a cell boundary relative to its neighbours so that the damping force is proportional to the rate of elongation of the cell (Kelvin dissipation).

Having constructed and solved the discrete model, we then use the methods of discrete-to-continuum upscaling to derive new PDE models using Taylor expansions local to each discrete cell, which requires that the properties of the individual cells (e.g. shear modulus) vary smoothly along

the array. The discrete and PDE models are solved computationally for a range of imposed boundary and growth conditions.

We demonstrate excellent agreement between the solutions (including diagnostics such as pressure and stretch along the array) of the discrete and PDE models for a number of examples, including a ring of cells (e.g. myocytes) with a wave of active contraction, growth of incompressible neo-Hookean cells, and stress-dependent growth. Qualitative differences are found in long-time scaling laws for the growth of the array of cells for stress-dependent and independent cell division rules. These methods provide a rational multiscale approach for deriving continuum models for soft tissues based on measured properties of individual cells. They can be extended to 2D and 3D.

This work was funded by the U.K. EPSRC SofTMech Centre for Mathematical Sciences in Healthcare (EP/N014642/1).

On the Figure of Elastic Planets: Gravitational Collapse and Infinitely Many Equilibria

Fei Jia[1,2], Ousmane Kodio[2], S. Jon Chapman[2], Alain Goriely[2,*]

[1]Department of Astronautical Science and Mechanics, Harbin Institute of Technology, No. 92 West Dazhi Street, Harbin 150001, China; [2]Mathematical Institute, University of Oxford, Oxford OX2 6GG, UK.

Abstract

A classic problem of elasticity is to determine the possible equilibria of an elastic planet modelled as a homogeneous compressible spherical elastic body subject to its own gravitational field. In the absence of gravity the initial radius is given and the density is constant. With gravity and for small planets, the elastic deformations are small enough so that the spherical equilibria can be readily obtained by using the theory of linear elasticity. For larger or denser planets, large deformations occur and the general theory of nonlinear elasticity is required to obtain the solution. Depending on the elastic model, we show that there may be parameter regimes where there exist no equilibrium or arbitrarily many equilibria. Yet, at most two of them are dynamically stable with respect to radial disturbances. In some of these models, there is a critical initial radius at which spherical solutions cease to exist. For planets with larger initial radii, there is no spherical solution as the elastic forces are not sufficient to balance the gravitational force. Therefore, the system undergoes gravitational collapse, an unexpected phenomenon within the framework of classical continuum mechanics.

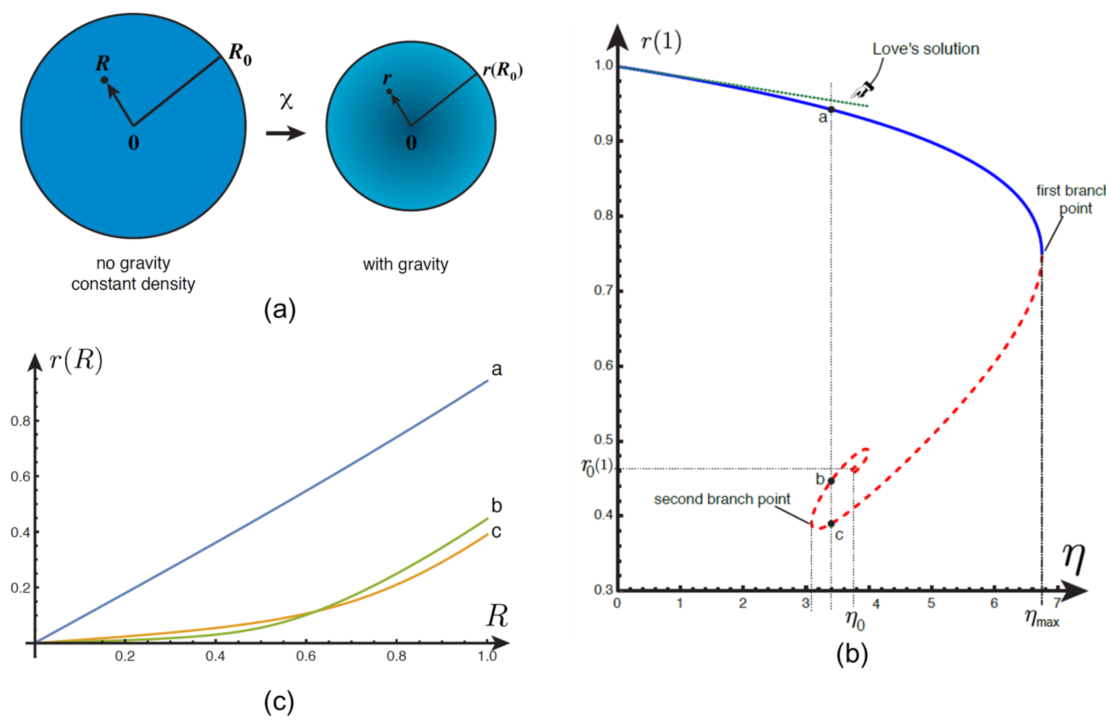


Figure 1. Spherical deformation of a spherical planet under its own gravity field: (a) sketch of the model, (b) current sphere radius as a function of increasing gravity η, (c) the three solution profile (a,b,c) $r(R)$ for $\eta = 3.4$.

Domain formations and pattern evolutions due to instabilities in soft composites

J. Li[2], N. Arora[1], V. Slesarenko[2], P. Galich[3], A. Goshkoderia[2],
Stephan Rudykh[1]

[1]Mechanical Engineering Dept., University of Wisconsin–Madison, USA
(rudykh@wisc.edu)
[2]Aerospace Engineering Dept., Technion–I.I.T., Haifa, Israel
[3]Materials Science and NanoEngineering Dept., Rice University, Houston, USA

We investigate the mechanical stability of soft microstructured materials subjected to finite deformations. The periodically structured materials exhibit the so-called buckling phenomenon when compressed to a critical level. For example, wavy patterns develop in layered materials upon achieving the critical compressive strain level. In this work, we perform a detailed study of the dependence of pattern formations on material properties and geometrical parameters of soft composites such as hyperelastic laminates [1], 3D fiber composites [2,3], and void-matrix-particles soft periodic systems [4]. The composite samples with various materials for the stiffer phase and matrix were manufactured by means of 3D printing. These 3D printed samples were mechanically deformed, and the critical buckling strain and post-bifurcation geometry were determined experimentally [1,3,4]. The experimental findings are in a good agreement with the predictions obtained from the numerical simulations. Moreover, we show that the buckled shapes in the laminates with rate dependent constituents can be tuned and controlled by the applied strain-rate [1]. In laminates, the critical wave-length of the instability induced wavy patterns varies with the applied strain-rate. It is possible, however, to design composites, in which the critical wavelength remains the same independent of the applied strain rate, while the critical strain varies with a change in the strain-rate. Thus, a wider range of wavy patterns with various wavelengths and amplitudes can be archived through the combination of the instability and viscoelastic phenomena [1]. Finally, we illustrate that, on the example of void-matrix-inclusion periodic system [4], that that instability induced pattern transformation can be used induced elastic wave band gaps through applied deformation [4].

Finally, recently discovered new type of instability-induced domain formations in soft composites will be presented [5].

References

[1] V. Slesarenko and S. Rudykh, *Harnessing viscoelasticity and instabilities for tuning wavy patterns in soft layered composites*. Soft Matter, **12**, 3677-3682 (2016)

[2] V. Slesarenko and S. Rudykh, *Microscopic and macroscopic instabilities in hyperelastic fiber composites*. Journal of the Mechanics and Physics of Solids, **99**, 471-482 (2017)

[3] J. Li, V. Slesarenko, P. Galich, and S. Rudykh, *Instabilities and pattern formations in 3D-printed deformable fiber composites*. Composites Part B, **148**, 114-122 (2018)

[4] J. Li, V. Slesarenko and S. Rudykh, *Auxetic multiphase soft composite material design through instabilities with application for acoustic metamaterials*. Soft Matter, **14**, 6171-6180 (2018)

[5] J. Li, T.D.Pallicity, V. Slesarenko, A. Goshkoderia and S. Rudykh, *Domain formations and Pattern Transitions via Instabilities in Soft Heterogeneous Materials* Advanced Materials, https://doi.org/10.1002/adma.201807309 (2019)

An efficient stochastic finite element framework for biological soft matter mechanics based on perturbation theory and Karhunen-Loève decomposition.

Georges Limbert[1,2*], Teja Melink[3] and Jože Korelc[3]

[1] national Centre for Advanced Tribology at Southampton (nCATS) | Bioengineering Science Research Group, Department of Mechanical Engineering, Faculty of Engineering and Physical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom

[2] Biomechanics and Mechanobiology Laboratory, Biomedical Engineering Division, Department of Human Biology, Faculty of Health Sciences, University of Cape Town, Anzio Road, Observatory 7925, South Africa

[3] Faculty of Civil and Geodetic Engineering, University of Ljubljana, Jamova cesta 2, 1000 Ljubljana, Slovenia

*Corresponding author: g.limbert@soton.ac.uk

Abstract

One of the significant challenges of research in biological soft matter, and more particularly in the field of biomedical sciences, is the integration of variability in predictive finite element models. Sources of variability include physical properties, geometry, boundary and loading conditions. Stochastic finite element analyses are notoriously more complex and computationally expensive than their deterministic counterparts. It is therefore advantageous to develop general, efficient and modular methodologies to conduct such analyses, especially those that can capture spatial variation of random variables (i.e. stochastic fields). Here, it is proposed to represent stochastic fields using the Karhunen-Loève (KL) decomposition and calculate the stochastic response of the system using a second-order sensitivity analysis within a unified finite element environment. KL decomposition is computed via the formulation of a macro-element associated with a Fredholm integral equation whose solution is obtained via standard Galerkin procedure. The stochastic finite element framework was assessed on toy examples and solutions compared to large-scale Monte-Carlo simulations. On some problems, the gain in computational speed afforded by the stochastic framework is over four orders of magnitude. The ability of a perturbation approach to capture propagation of uncertainties in highly non-linear systems is inherently limited by the expansion order (here, second order). However, the use of automatic differentiation combined with optimisation of computer code means that extension of the stochastic finite element framework to higher orders is possible and should be explored in the future.

Computational modelling of phase-transforming magnetic solids

Andreas Menzel[1,2], Thorsten Bartel[1], Björn Kiefer[3]

[1] Institute of Mechanics, Department of Mechanical Engineering, TU Dortmund, Leonhard-Euler-Str. 5, 44227 Dortmund, Germany

[2] Division of Solid Mechanics, Department of Construction Sciences, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden

[3] Institute of Mechanics und Fluid Dynamics, Chair of Applied Mechanics - Solid Mechanics, TU Bergakademie Freiberg, Lampadiusstr. 4, 09596 Freiberg, Germany

In this contribution, we present a variational framework which is suitable for canonical finite element implementations of material models for multiferroic materials. The specific magnetomechanically coupled material model builds on a micromechanically motivated parametrisation of the underlying microstructure and the modelling of its evolution in terms of reorientation processes ("variant switching"), domain wall motion, and rotations of vectors which accounts for the spontaneous magnetisations in each domain. The applied homogenisation procedure for the definition of different states in each of the coexisiting material's phases relies on mathematically sound energy relaxation methods using first order laminates.

The difficulty of implementing such constitutive models into finite element codes lies in the fact that the magnetostatic energy stored in the demagnetisation field is non-local in nature. In other words, the microstructure evolution at a material point depends on the solution of the boundary value problem in the entire domain. Therefore, the underlying microstructural state variables are spatially fully resolved and discretised within the global FE framework. In line with [1], magnetomechanically coupled boundary value problems are used as numerical examples highlighting the capabilitites of the proposed framework e.g. in terms of the prediction of inhomogeneous magnetisation fields for arbitrarily shaped magnetic shape memory alloy specimens.

[1] K. Buckmann, B. Kiefer, T. Bartel, A. Menzel. Simulation of magnetised microstructure evolution based on a micromagnetics-inspired FE framework: application to magnetic shape memory behaviour, Arch. Appl. Mech., doi: 10.1007/s00419-018-1482-7, 2018

Effective balance equations for poroelastic composites

Laura Miller[1], Raimondo Penta[1]

[1] School of Mathematics and Statistics, University of Glasgow, UK

E-mail contacts: l.miller.3@research.gla.ac.uk; Raimondo.Penta@glasgow.ac.uk

ABSTRACT

We derive the quasi-static governing equations for the macroscale behaviour of a linear elastic porous composite comprising a matrix interacting with inclusions and/or fibers, and an incompressible Newtonian fluid flowing in the pores.

We assume that the size of the pores (*the microscale*) is comparable with the distance between adjacent subphases, and is much smaller than the size of the whole domain (*the macroscale*).

We then decouple spatial scales embracing the asymptotic (periodic) homogenization technique to derive the new macroscale model by upscaling the fluid-structure interaction problem between the elastic constituents and the fluid phase.

The resulting system of partial differential equations is of poroelastic-type, and encodes the properties of the microstructure in the coefficients of the model, which are to be computed by solving appropriate cell problems which reflect the complexity of the given microstructure.

The model reduces to the limit case of simple composites [III] when there are no pores, and standard Biot's poroelasticity [I, II] whenever only the matrix-fluid interaction is considered. We further prove rigorous properties of the coefficients, namely (a) major and minor symmetries of the effective elasticity tensor, (b) positive definiteness of the resulting Biot's modulus, and (c) analytical identities which allow us to define an effective Biot's coefficient.
This model is applicable when the interactions between multiple solid phases occur at the porescale, as in the case of various systems such as biological aggregates, constructs, bone, tendons, as well as rocks and soil.

REFERENCES

[I] Burridge, Robert, and Joseph B. Keller. "Poroelasticity equations derived from microstructure." The Journal of the Acoustical Society of America 70.4 (1981): 1140-1146.

[II] Dehghani, H., R. Penta, and J. Merodio. "The role of porosity and solid matrix compressibility on the mechanical behavior of poroelastic tissues." *Materials Research Express* 6.3 (2018): 035404.

[III] Penta, R. and Gerisch, A., 2017. The asymptotic homogenization elasticity tensor properties for composites with material discontinuities. Continuum Mechanics and Thermodynamics, 29(1), pp.187-206.

What the cornea shape can tell about collagen microstructure and stress distribution

Anna Pandolfi, Andrea Montanino

Politecnico, Piazza Leonardo da Vinci 32, 20133 Milano, Italy, anna.pandolfi@polimi.it

Maurizio Angelillo

Universita' di Salerno, Via Giovanni Paolo II 32, Fisciano, Italy, mangelil@unisa.it

Alessio Gizzi

Universita' Campus Biomedico, Via A. del Portillo 21, Roma, Italy, a.gizzi@unicampus.it

Maurizio Vasta

Universita' di Pescara Chieti, Viale Pindaro 42, Pescara, Italy, mvasta@unich.it

We try to motivate mechanically the development of anomalous corneal shapes by using two simple approaches. The first one is based on a micromechanical model of the reinforcing collagen structure; the second one is based on the analysis of the stress state using only equilibrium.
First, we describe a simplified micromechanical model of the collagen fibrils that accounts for crosslink bonds. The reinforcing structure is modeled with two sets of parallel fibrils, connected by transversal bonds within the single fibril family and across the two families. The particular design chosen for this ideal structure relies on the fact that its ability to sustain loads depends on the degree of the crosslink, thus on the bond density and stiffness. The mechanical response of the system is analyzed according to the level of interlacing and on the bond stiffness. The weakening of transversal bonds is associated to an increase of the deformability of the system. In particular, the localized deterioration of transversal bonds due to mechanical, chemical, or enzymatic reasons can justify the weakening of the stromal tissue resulting in localized thinning and bulging typically observed in keratoconus [1].
Second, we model the human cornea as a statically determined membrane. A numerical investigation over 40 patient specific corneas (20 normal, 20 ectatic) is carried out to establish the relationship between the physiological geometry and distribution of the stresses, and to assess the possibility to obtain information on the stress state based only on topographic images. Comparative analyses reveal that, with respect to the normal corneas, in unhealthy corneas the pattern of the principal stress lines is modified markedly showing a deviation from the principal orientation of the collagen fibrils. The rotation of the principal stresses with respect to the fibril orientation is responsible of the transmission of a large amount of shear stresses onto the elastin-proteoglycan matrix. The anomalous loading of the matrix could be correlated to the evolution of time dependent shape modifications leading to ectasia [2].

References

[1] A. Pandolfi, A. Gizzi, M. Vasta. A microstructural model of crosslink interaction between collagen fibrils in the human cornea, PTA (in press), 2019.

[2] M. Angelillo, A. Montanino, and A. Pandolfi. An interpretation of the connection between collagen fibril microstructure and statically determined principal stress line distribution in the human cornea, 2018. Submitted.

Poroelasticity derived via asymptotic homogenization. State of the art and further perspectives

Raimondo Penta[1]

[1]School of Mathematics and Statistics, University of Glasgow, UK

E-mail contact: Raimondo.Penta@glasgow.ac.uk

ABSTRACT

The mechanical behavior of a solid elastic structure interplaying with fluid percolating
its pores can be studied via the Theory of Poroelasticity. There exists a large variety of scenarios of interest that can be treated by means of a poroelastic modeling approach, including soil mechanics, (bio) artificial constructs and biological tissues, such as bone, organs, healthy and malignant (tumorous) cell aggregates. Here, we revisit the equations of poroelasticity derived via asymptotic homogenization [I] and focus on recent theoretical and computational [II] advances on the subject.

REFERENCES

[I] Burridge, Robert, and Joseph B. Keller. "Poroelasticity equations derived from microstructure." The Journal of the Acoustical Society of America 70.4 (1981): 1140-1146.

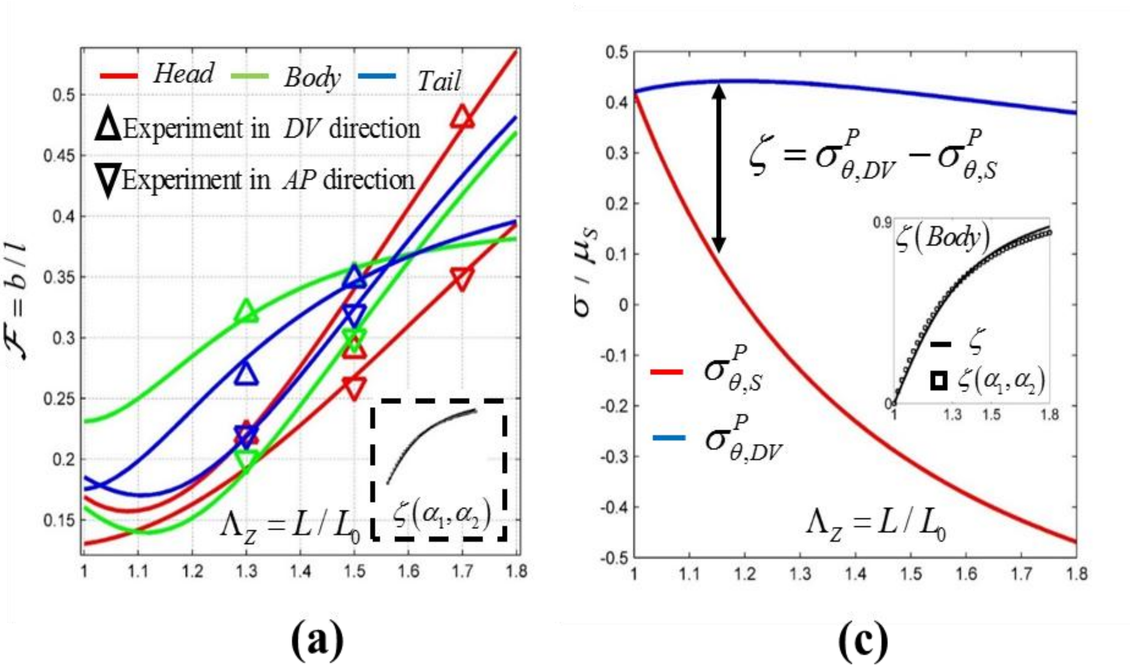
[II] Dehghani, H., R. Penta, and J. Merodio. "The role of porosity and solid matrix compressibility on the mechanical behavior of poroelastic tissues." *Materials Research Express* 6.3 (2018): 035404.

Assessing the Contribution of Active and Passive Stresses in C. elegans Elongation

Martine Ben Amar[1], **Paul Qiuyang-Qu**[1],Thanh Vuong-Brender[2], Michel Labouesse[3]

[1]Lab. de Physique of ENS and Sorbonne Université (Paris) ; [2] MRC, Crick Institute, (Cambridge, UK) ; [3] IBPS, Sorbonne Université, (Paris).

The role of the actomyosin network is investigated in the elongation of C. elegans during embryonic morphogenesis. We present a model of active elongating matter that combines prestress and passive stress in nonlinear elasticity. Using this model we revisit recently published data from laser ablation experiments to account for why cells under contraction can lead to an opening fracture. By taking into account the specific embryo geometry, we obtain quantitative predictions for the contractile forces exerted by the molecular motors myosin II for an elongation up to 70% of the initial length. This study demonstrates the importance of active processes in embryonic morphogenesis and the interplay between geometry and nonlinear mechanics during morphological events. In particular, it outlines the role of each connected layer of the epidermis compressed by an apical extracellular matrix that distributes the stresses during elongation.



(a) Crack opening in Seam cells for head, body and tail in different (DV and AP) directions with shape factor $F_i = 2\left(\sigma_i^P + \zeta_i\right)/E_i$. (c) Active stress ζ evaluated as the difference of passive stress σ_θ^P between DV and Seam cells.[1]

[1] Email: benamar@lps.ens.fr ; qiuyangqu@gmail.com; vuongthikimthanh@gmail.com ; michel.labouesse@sorbonne-universite.fr

An asymptotic homogenization approach for modelling the microscopic evolution of heterogeneous media

Ariel Ramírez-Torres [a], Salvatore Di Stefano [a], Alfio Grillo [a], Reinaldo Rodríguez-Ramos [b], José Merodio [c], Raimondo Penta [d]

[a] Dipartimento di Scienze Matematiche ‘‘G. L. Lagrange’’, Politecnico di Torino, Torino, 10129, Italy. ariel.ramirez@polito.it, salvatore.distefano@polito.it, alfio.grillo@polito.it
[b] Departamento de Matemáticas, Facultad de Matemática y Computación, Universidad de La Habana, La Habana, CP 10400, Cuba. reinaldo@matcom.uh.cu
[c] Departamento de Mecánica de los Medios Continuos y T. Estructuras, E.T.S. de Caminos, Canales y Puertos, Universidad Politécnica de Madrid, Madrid, CP 28040, Spain. merodioj@gmail.com
[d] School of Mathematics and Statistics, Mathematics and Statistics Building, University of Glasgow, University Place, Glasgow G12 8QQ, UK. Raimondo.Penta@glasgow.ac.uk

In this work we study the dynamics of an inhomogeneous and heterogeneous medium with evolving microstructure. With this scope, we take advantage of the two-scale asymptotic homogenization technique, which permits to formulate the original problem in an equivalent homogenized problem describing the overall behavior of the medium, and the auxiliary-local problems, which take into account the microscopic interactions. We interpret the evolution of the medium’s internal structure with the production of plastic-like distortions, and we describe it by using the Bilby-Kröner-Lee decomposition of the deformation gradient tensor in a scale-dependent fashion. In particular, the evolution law for the tensor of plastic-like distortions is assumed to obey a phenomenological flow rule driven by stress. After a general analysis, the study is framed in the limit of small deformations with large plastic-like distortions and numerical results are presented.

[1] D. Cioranescu, P. Donato, *An Introduction to Homogenization*, Oxford University Press Inc., New York, EE. UU, 1999.

[2] M.V. Mićunović, *Thermomechanics of Viscoplasticity - Fundamentals and Applications*, Springer, Heidelberg, Germany, 2009.

[3] A. Grillo, R. Prohl, G. Wittum, *A poroplastic model of structural reorganisation in porous media of biomechanical interest*, Contin. Mech. Thermodyn. 28 (2016) 579-601.

[4] A. Ramírez–Torres, S. Di Stefano, A. Grillo, R. Rodríguez–Ramos, J. Merodio, R. Penta, *An Asymptotic Homogenization Approach to the Microstructural Evolution of Heterogeneous Media*, Int. J. Non-Lin. Mech. 106 (2018) 245-257.

Macroscopic thermal profile of heterogeneous cancerous breasts. A multiscale three dimensional analysis

Reinaldo Rodríguez-Ramos[1], Jorge Marchena-Menéndez[1], Ariel Ramírez-Torres[2], Raimondo Penta[3] and José Merodio[4]

[1] Departamento de Matemáticas, Facultad de Matemática y Computación, Universidad de La Habana, 10400, La Habana, Cuba

[2] Dipartimento di Scienze Matematiche (DISMA) ''G.L. Lagrange'', Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129, Torino, Italy

[3] School of Mathematics and Statistics, Mathematics and Statistics Building, University of Glasgow, University Place, Glasgow G12 8QQ, UK

[4] Departamento de Mecánica de los Medios Continuos y T. Estructuras, E.T.S. de Caminos, Canales y Puertos, Universidad Politécnica de Madrid, Madrid, 28040, España

Abstract: The present work focuses on the multiscale analysis of temperature maps for cancerous breasts. A three-dimensional model is proposed based on a system of bio-heat transfer equations for the healthy and cancerous breast regions, which are characterized by different microstructure and thermo-physical properties. The geometrical model of the cancerous breast is identified by the presence of muscle, glandular and fat tissues, as well as the heterogeneous tumor tissue. The latter is assumed to be a two-phase periodic composite with a spherical inclusion. A cubic lattice distribution is chosen, wherein the constituents exhibit isotropic thermal conductivity behavior. The tissue effective thermal conductivities are computed by means of the asymptotic homogenization approach, i.e. by solving relevant periodic problems on the cell, which is representative of the malignant tissue microstructure. These are then exploited to solve the macroscale homogenized model by finite elements. The obtained results in terms of temperature maps are successfully compared with relevant experiments and could pave the way towards the development of a robust multiscale mathematical framework featuring microstructural information, e.g. given by the microvasculature, which can be useful in cancer diagnosis.

Emergence of new Willis couplings in responsive metamaterials

René Pernas-Salomón[1] and Gal Shmuel[2]
Faculty of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel

Metamaterials posses microstructure designed to acquire properties not found in nature. An epitome in acoustics and solid mechanics is Willis coupling, which refers to the particle velocity-stress coupling, and of great significance since it controls mechanical waves. We here reveal new couplings, analogous to Willis coupling, when considering patterns of materials that mechanically interact with magnetic, electric or thermal fields. To this end, we develop a rigorous homogenization method for the effective properties of such responsive meta- materials. As an example, we apply the scheme to piezoelectric materials, and unveil coupling of the velocity and electric fields. Hence, Willis-like couplings in responsive metamaterials open new avenues for active wave control by modulation of external stimuli.

[1]rpernas@me.technion.ac.il
[2] meshmuel@technion.ac.il

Bulging Bifurcation and post-bifurcation of an inflated and extended residually-stressed tube and aneurysms formation and propagation

Carlos de los Santos, Dariel Desena, Hamidreza Dehghani and Jose Merodio

Department of Continuum Mechanics and Structures, Technical University of Madrid, Madrid, Spain

The problema at hand is the combined extension and inflation of a circular cylindrical tube subject to radial and circumferential residual stresses. A numerical procedure to analyze bifurcation and post-bifurcation of a finite deformation boundary-value problem for a residually-stressed elastic body is studied. The material model, given by a residual-stress dependent nonlinear elastic constitutive law in terms of invariants, is implemented in a finite element code. The numerical procedure is developed based on the modified Riks method.The dependence of bifurcation and post-bifurcation behavior of tubes under the loading at hand on residual stresses is shown and compared with results when there is no residual stress. The finite deformation boundary-value problem is described mainly in terms of the inflation pressure, as well as the axial and azimuthal stretches of the tube. The dependence of these quantities on bifurcation is illustrated graphically for different values of the parameters (in dimensionless form) involved, in particular, the strength of the residual stress. It is found that bulging bifurcation is expected for sufficiently large values of the axial stretch.

New frontiers in residual stress modelling

Taisiya Sigaeva (taisiya.sigaeva@ucalgary.ca) and Elena S. Di Martino (edimarti@ucalgary.ca)

Schulich School of Engineering, University of Calgary, Canada

Residual stresses are vital in modelling in vivo behavior of arteries. However, current approaches to measure residual deformations and to estimate residual stresses analytically face certain limitations and challenges. In our work, we investigate two acute problems related to residual stress in arteries and discuss some important physiological implications ensuing therefrom.

Anisotropic residual stresses. Each layer in the artery has its unique mechanical properties and behaves kinematically differently upon release of residual stresses. This is due to the unique microstructure of the layers and their response to applied loads. Furthermore, the microstructure also governs the mechanical anisotropy of arteries, even at smaller strains. Therefore, it is important to treat residual stresses as an anisotropic phenomenon. In this work, we model residual deformation in all three layers of a healthy abdominal aorta using the three-dimensional model from [1]. However, in contrast to [1], we take into account each layer's unique mechanical and structural properties via the anisotropic material model from [2]. We then compare residual stresses computed assuming isotropy vs. anisotropy, thus quantifying the contribution of the microstructure to the residually deformed unloaded state.

Novel analytical method to handle asymmetric openings. The conventional experiment to release residual stresses in arteries is the opening angle method, in which an aortic ring is cut axially allowing it to spring-open. In some tissues, especially pathological ones, the resulting residual deformations are highly asymmetric and irregular, making the quantification of residual stresses by analytical model impractical. As a consequence many studies limit the quantification of residual stresses to rings that open in a conventional symmetric fashion. We developed an approach that can handle these non-trivial openings – opened rings can be digitally split into multiple sectors and the corresponding residual stresses can be calculated for the individual sectors and then averaged; we call this approach the multi-sector method. The proposed method is demonstrated on two samples with abrupt curvature changes in the stress-free configuration. For simplicity we modelled them as homogeneous neo-Hookean materials and showed that the multi-sector method predicts higher residual stresses than the classical opening angle method.

Considering that even small changes in the residual stresses significantly affect the in-vivo stress state of arteries, more accurate methods of residual stress estimation as discussed herein may be vital in development of stress-based risk assessment tools.

The first project is a joint work with Dr Gerhard Sommer and Dr Gerhard A. Holzapfel (Institute of Biomechanics, Graz University of Technology); the second project is a joint work with Dr Michel Destrade (NUI Galway). This research has been made possible by a James M. Flaherty Research Scholarship from the Ireland Canada University Foundation, with the assistance of the Government of Canada/avec l'appui du gouvernement du Canada. Contributions from National Science and Engineering Research Council of Canada (NSERC) Discovery Grant and the Heart and Stroke Foundation of Canada are gratefully acknowledged.

[1] Holzapfel GA et al. (2010). Journal of the Royal Society Interface, 5; p. 20090357.

[2] Holzapfel GA et al. (2015). Journal of the Royal Society Interface, 12(106); p. 20150188

On new challenges in polar elasticity of fibre-reinforced materials

Kostas P. Soldatos

University of Nottingham, School of Mathematical Sciences, Nottingham NG7 2RD, UK

Abstract

This talk is biased by the fact that non-linear elasticity of fibre-reinforced materials was essentially nurtured and developed through the research activity and effort of a group of Nottingham-based mathematicians. It aims to outline, briefly and lightly, theoretical features and principal relevant challenges that became known gradually over the years, assisted, and continue to assist the development of the theory. A short historical overview begins with the conventional, non-polar version of the theory, and proceeds towards its more recent, polar hyperelasticity counterpart where fibres possess thickness and bending stiffness. Reference is made to mathematical modelling challenges that, although old in fibre-reinforced material mechanics, seem to become now better understood or manageable. Attention is also given to the constitutive equations of the relevant linearised version of the theory, and to the subsequent fact that, unlike their conventional elasticity counterparts, the governing equations of the present, polar elasticity theory are not elliptic even within the small/infinitesimal deformation regime. Some new results stemming from the latter observation are presented. These are complemented by new relevant challenges which are put forward for discussion.

Pattern evolution in bending dielectric-elastomeric bilayers

Yipin Su su_yp@zju.edu.cn. School of Mathematics, Statistics and Applied Mathematics, NUI Galway, University Road, Galway, Ireland

Bin Wu wubinlongchang@163.com, Department of Mechanical and Aerospace, Politecnico di Torino, Turin, Italy

Weiqiu Chen chenwq@zju.edu.cn, Department of Engineering Mechanics,

Zhejiang University, Hangzhou 310027, P.R. China

Michel Destrade michel.destrade@nuigalway.ie, School of Mathematics, Statistics and Applied Mathematics, NUI Galway, University Road, Galway, Ireland

We propose a theoretical analysis of smart bending deformation of a dielectric-elastic bilayer in response to a voltage, based on the nonlinear theory of electro-elasticity and the associated linear incremental field theory. We reveal that the mechanism allowing the bending angle of the bilayer can be tuned by adjusting the applied voltage. Further, we investigate how much can the bilayer be bent before it loses its stability by buckling when one of its faces is under too much compression. We find that the physical properties of the two layers must be selected to be of the same order of magnitude to obtain a consequent bending without encountering buckling. If required, the wrinkles can be designed to appear on either the inner or the outer bent surface of the buckled bilayer. We validate the results through comparison with those of the classical elastic problem.

Keywords: dielectric-elastic bilayer, smart bending, electroelasticity, incremental theory, buckling

Morphoelastic remodeling of collagenous fibers under cyclic loading

Heiko Topol[(a)], Hasan Demirkoparan[(a)], Thomas J. Pence[(b)]

[(a)] Carnegie Mellon University Qatar, P.O. Box 24866, Doha, Qatar (htopol@qatar.cmu.edu, hasand@qatar.cmu.edu)
[(b)] Department of Mechanical Engineering, Michigan State University, East Lansing, MI 48824-1226, USA (pence@egr.msu.edu)

Soft tissues undergo remodeling processes, which are related to numerous biological, chemical and physical processes. Different experiments such as [1] find that enzymatic collagen fiber dissolution is slowed down when the fibers are stretched. This effect has been taken into account in a continuum mechanics framework by Demirkoparan et al. [2]. In this continuum mechanics framework the mechanical properties of the fibers have been modeled in terms of the strain energy density of basic fiber entities, so-called protofibers, and a fiber survival kernel, which describes the development of the fiber density with the deformation history of the material. This kernel is modeled in terms of a constant fiber creation rate and a mechano-sensitive fiber dissolution rate, which decreases with the amount of fiber stretch. The ground substance matrix of the material is taken to be an incompressible neo-Hookean solid, which despite its simplicity may provide a sufficient accuracy in describing the ground substance properties.

The natural or undeformed configuration of the fibers may differ from the natural configuration of the ground substance matrix. In different loading scenarios such as swelling [3] and cylindrical inflation [4] it has been shown that the choice of the natural fiber configuration has a crucial impact on the development of the fiber properties.

In our work we study the development of relation between loading and deformation of the material for different fiber natural configurations, which may results into stiffening or softening of the material with cyclic loading. The fibers natural configuration may coincide with the natural configuration of the surrounding ground substance matrix, or the fiber may be synthesized in a state of pre-stretch.

Acknowledgement: This work was made possible by NPRP Grant No.8-2424-1-477 from the Qatar National Research Fund (a member of the Qatar Foundation). The statements made herein are solely the responsibility of the authors.

[1] A. P. Bhole, B.P. Flynn, M. Liles, N. Saeidi, C. Dimarzio, J. Ruberti, Philos. Trans. R Soc. Lond. A 367: 3339- 3362 (2009)

[2] H. Demirkoparan, T.J. Pence, A. Wineman, Int. J. Nonlinear Mech. 56:86–93 (2013)

[3] H. Topol, K. Gou, H. Demirkoparan, T.J. Pence, Biomech. Model.Mechan. 17:1543-1567 (2018)

[4] H. Topol, H. Demirkoparan, T.J. Pence, Eur. J. Mech. A – Solid (submitted) (2019)

Explicit transfer matrices of elastic layers and applications

Pham Chi Vinh,

Faculty of Mathematics, Mechanics and Informatics

Hanoi University of Science, 334 Nguyen Trai Str., Thanh Xuan, Hanoi, Vietnam.

In this report, we present the derivation of transfer matrices in explicit form of orthotropic and pre-stressed elastic layers. Since these transfer matrices are totally explicit, they are convenient tools for solving various practical problems of wave propagation in layered elastic media. To prove this, we employ the obtained explicit transfer matrices to investigate the reflection and transmission of plane waves at orthotropic and pre-stressed elastic layers, and the propagation of Rayleigh waves in elastic half-spaces coated by orthotropic and pre-stressed elastic layers. The formulas in closed form for reflection and transmission coefficient and the explicit dispersion equations in explicit form are derived quickly by using the obtained explicit transfer matrices.

A semi-analytical procedure for the homogenisation of dielectric laminated composites at finite strains

Valentina Volpini[1], Lorenzo Bardella[1], Massimiliano Gei[2]

[1]*Department of Civil, Environmental, Architectural Engineering and Mathematics, University of Brescia, Brescia, Italy*
E-mail:v.volpini@unibs.it,lorenzo.bardella@unibs.it

[2]*School of Engineering, Cardiff University, Cardiff, UK*
E-mail: *geim@cardiff.ac.uk*

Keywords: dielectric laminated composites; electromechanical actuation; homogenisation

We focus on the layout optimisation of hierarchical laminated composites that can be profitably employed to enhance the actuation performance of electrostatically-activated soft dielectric transducers [1]. In particular, we study the behaviour of a rank-two laminate constituted by ideal dielectric phases obeying nonlinear elasticity. This actuation response is evaluated by coupling two rank-one problems: the first one is concerned with the two-phase microstructure constituting the so-called core; the second one, at the mesoscopic scale, involves the core and a third homogeneous phase. Concerning the microscale rank-one problem, we adopt for the core the analytical form of the effective free energy density obtained by Spinelli and Lopez-Pamies [2] for isochoric neo-Hookean elasticity. Hence, we solve the whole problem by imposing the macroscopic boundary prescriptions and the electro-mechanical continuity conditions at the laminate interfaces.
We analytically simplify at lowest terms the resulting set of nonlinear equations, thus obtaining a partly uncoupled system, which, as a main novelty with respect to the literature, is unaffected by the local Lagrangian multipliers ensuing from the assumed isochoric deformation. We demonstrate the computational efficiency of our procedure by studying two different composites previously investigated in [1, 3]. Moreover, on the basis of sensitivity analyses with respect to the micro- and meso-structural parameters, we provide new layouts able to optimise the actuation stretch.

References

[1] Tian, L., Tevet–Deree, L., deBotton, G. and Bhattacharya, K., Dielectric elastomer composites, J. Mech. Phys. Solids, 60, 181-198 (2012).
[2] Spinelli, S.A. and Lopez-Pamies, O., Some simple explicit results for the elastic dielectric properties and stability of layered composites, Int. J. Eng. Sci., 88, 15-28 (2015).
[3] Gei, M. and Mutasa, K.C.K., Optimisation of hierarchical dielectric elastomer laminated composites, Int. J. Nonlinear Mech., 106, 266-273 (2018).

Smooth waves and shocks of finite amplitude in soft materials

Ron Ziv (ronz@campus.technion.ac.il)

Gal Shmuel (meshmuel@technion.ac.il)

Faculty of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel

Recently developed soft materials exhibit nonlinear wave propagation with potential applications for energy trapping, shock mitigation and wave focusing. We address finitely deformed materials subjected to combined transverse and axial impacts, and study the resultant nonlinear waves. We determine the dependency of the induced motion on the impact, pre-deformation and the employed constitutive models. We analyze the neo-Hookean constitutive model and show it cannot capture shear shocks and tensile-induced shocks, in contrast with experimental results on soft materials. We find that the Gent constitutive model predicts that compressive impact may not be sufficient to induce a quasi-pressure shock - yet it may induce a quasi-shear shock, where tensile impact can trigger quasi-pressure shock - and may simultaneously trigger a quasi-shear shock, in agreement with experimental data. We show that the tensile impact must be greater than a calculated threshold value to induce shock, and demonstrate that this threshold is lowered by application of pre-shear [1].

References

[1] R. Ziv and G. Shmuel. Smooth waves and shocks of finite amplitude in soft materials. *Under review*, 2019. URL https://arxiv.org/abs/1901.00529

The nonlinear mechanics of surface growth

Giuseppe Zurlo, *Lev Truskinovsky*

Abstract: *In this work we present the nonlinear theory of surface growth, where mass is being continuously deposited on the boundary surface of a growing body. Our focus is on the mechanical aspects of this process, and we establish a precise connection between the controls at deposition and the development of residual stresses during the process. In particular, we shed light on the non-local character of the process. This new work [1] generalises some previous works [2],[3] on the linear theory of surface growth, and it highlights the importance of a geometrically non-linear setting, in particular to deal with biologically relevant examples such actin polymerization against rigid walls.*

[1] Truskinovsky L., Zurlo G.; Nonlinear elasticity of incompatible surface growth, Phys. Rev. E 99, 053001 (2019)

[2] Zurlo G., Truskinovsky L.; Inelastic surface growth, Mechanics Research Communications 93,174–179 (2018)

[3] Zurlo G., Truskinovsky L.; Printing Non-Euclidean Solids, Phys. Rev. Lett. 119, 048001 (2017)

PARTICIPANTS

Nir Alboteanu niralbo@post.bgu.ac.il

Martine Ben Amar **benamar@lps.ens.fr**

Valentina Balbi v.balbiv@gmail.com

Davide Bigoni davide.bigoni@unitn.it

Eliana Bortot eliana.bortot@gmail.com

Gal Debotton debotton@bgu.ac.il

Shawn A. Chester **shawn.a.chester@njit.edu**

Noy Cohen **noyco@technion.ac.il**

Hannah Conroy Broderick *hannah.conroybroderick@nuigalway.ie*

Hui-Hui Dai mahhdai@cityu.edu.hk

Hamidreza Dehghani dehghanihamid2@gmail.com

Michel Destrade michel.destrade@nuigalway.ie

Włodzimierz Domański domanski.wlodek@gmail.com

Luis Dorfmann Luis.Dorfmann@tufts.edu

Yangkun Du duyangkunzju@gmail.com

Elliot L. Elson elson@wustl.edu

Marcelo Epstein mepstein@ucalgary.ca

Aleksander Franus *a.franus@il.pw.edu.pl*

Massimiliano Gei *geim@cardiff.ac.uk*

Giulio G. Giusteri giulio.giusteri@math.unipd.it

Alfio Grillo alfio.grillo@polito.it

Nick Hill Nicholas.Hill@glasgow.ac.uk

Gerhard A. Holzapfel gerhard.holzapfel@ntnu.no

Stanisław Jemioło s.jemiolo@il.pw.edu.pl

Fei Jia jiafei0009@gmail.com

Georges Limbert g.limbert@soton.ac.uk

Xiaoyu Luo Xiaoyu.Luo@glasgow.ac.uk

Elena S. Di Martino edimarti@ucalgary.ca

Andreas Menzel andreas.menzel@udo.edu

Jose Merodio merodioj@gmail.com

Laura Miller l.miller.3@research.gla.ac.uk

Francisco Javier Montans <Fco.Montans@upm.es>

Raymond Ogden Raymond.Ogden@glasgow.ac.uk

Anna Pandolfianna.pandolfi@polimi.it

Raimondo Penta raimond46@gmail.com

Qiuyang Qu qiuyangqu@gmail.com

Ariel Ramírez Torres arielrt02@gmail.com

Reinaldo Rodriguez rerora2006@gmail.com

Stephan Rudykh rudykh@wisc.edu

Gal Shmuel meshmuel@technion.ac.il

Taisiya Sigaeva taisiya.sigaeva@ucalgary.ca

Kostas Soldatos etzkps@maths.nottingham.ac.uk

Yipin Su yipin.su@nuigalway.ie

Heiko Topol heiko.topol@gmail.com

Pham Vinh pcvinh@gmail.com

Valentina Volpini *v.volpini@unibs.it*

Ron Ziv ronz@campus.technion.ac.il

Giuseppe Zurlo giuseppe.zurlo@nuigalway.ie