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PROPAGATION OF THERMOELASTIC WAVES IN LAYERED STRUCTURES

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SOMMARIO
Si studia la propagazione di onde termoelastiche di piccola ampiezza in un mezzo multilaminato a temperatura costante sottoposto a deformazione finita. Il comportamento del materiale è descritto dalla funzione energia libera di Helmholtz ottenuta dalla teoria entropica modificata. Al fine di evitare instabilità numeriche si è adottata una particolare tecnica che elimina, nelle equazioni, i termini esponenziali divergenti.

ABSTRACT
Thermoelastic small-amplitude wave propagation in laminated structures subject to an imposed pre-strain, at constant temperature, is analysed. The material is described by the Helmholtz free-energy function within the modified entropic theory. A specific numerical technique is adopted to avoid numerical instabilities.

1. INTRODUCTION

Thermal effects represent a primary origin of failure for a broad class of multilayered devices. Due to different mechanical and thermal properties between constituents, temperature changes can introduce residual stresses which may lead to interface debonding in ductile materials, or enhance crack formation in brittle [1]. As a consequence, a number of mechanical models have been proposed to estimate the importance of thermal loadings in the analyses of instability and vibration of laminated structure [2, 3, 4]. All these models are however based on approximate plate theories, while our interest is to develop a framework for the analysis of small-amplitude waves superimposed upon an arbitrarily large and uniform deformation of a nonlinear elastic material. This framework was thoroughly analysed in the isothermal case for quasi-static deformation [5, 6, 7, 8, 9], while isothermal vibrations were considered mainly by Ogden and co-Workers [10, 11].

Within the framework of modified entropic theory, layered and compressible hyperelastic, nonlinear materials are considered in this article, deformed an arbitrary amount with deformations having principal Eulerian axes aligned parallel and orthogonal to the layers. Temperature is assumed uniform in this configuration and equal in all layers. Thermoelastic, plane strain, and small-amplitude waves are analysed from this state, in a fully coupled formulation.

2. GOVERNING EQUATIONS

An elastic (compressible) structure made up of different layers is considered, subject to uniform (arbitrary large) deformation and temperature. In the reference configuration, the layers are rectangular strips of infinite length in the direction 1 and 3 with material points defined by the coordinates (for the h-th layer)

$$-\infty < x_i^0 < +\infty \quad (i = 1, 3), \qquad x_2^{0uh} < x_2^0 < x_2^{0lh}, \tag{1}$$

where u and l stand for 'upper' side and 'lower' side respectively. The layers are subject to a given temperature Θ and to uniform stretch in directions 1 and 3, so that the current configuration (see Fig. 1) is defined by

$$-\infty < x_i < +\infty \quad (i = 1, 3), \qquad x_2^{uh} < x_2 < x_2^{lh}, \tag{2}$$

where $x_i = \lambda_i x_i^0$ $(i = 1, 3)$ and $x_2 = x_2^{uh} + \lambda_2(x_2^0 - x_2^{0uh})$, being λ_i $(i = 1, 2, 3)$ the principal stretches. Note that even if λ_1, λ_3 are imposed for the whole structure, λ_2 may differ between layers since it depends on the elastic constants of the examined stratum.

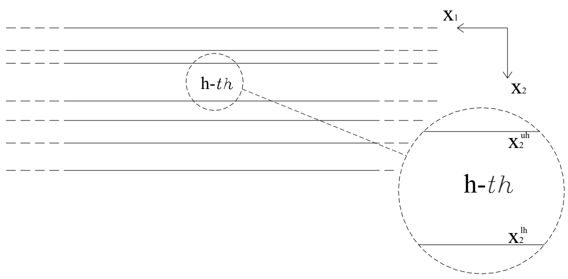

Figure 1: The multilayer structure.

To describe the constitutive behaviour of the layer material the Helmholtz free energy ψ, defined in terms of the deformation gradient F_{ij} and the absolute temperature Θ, is introduced, so that the first Piola-Kirchhoff stress tensor S_{ij} and the entropy η are given by

$$S_{ij} = \frac{\partial \psi(F_{kl}, \Theta)}{\partial F_{ij}}, \quad \eta = -\frac{\partial \psi(F_{kl}, \Theta)}{\partial \Theta}. \tag{3}$$

As a consequence, the specific internal energy e follows from the relationship $e = \psi + \Theta\eta$.

For an isotropic material obeying the modified entropic theory [12] the following neo-Hookean form for ψ will be adopted:

$$\begin{aligned}\psi \;=\; & \frac{1}{2}\left[\lambda_0 \left(J-1\right)^2 + \mu_0 \left(\lambda_1^2+\lambda_2^2+\lambda_3^2-3-2\ln J\right)\right]\Theta/\Theta_0 \\ & -3\alpha_0\kappa_0(J-1)(\Theta-\Theta_0)+c[\Theta-\Theta_0-\Theta\ln(\Theta/\Theta_0)],\end{aligned} \tag{4}$$

where λ_0, μ_0 and $\kappa_0 = \lambda_0 + 2\mu_0/3$ are, respectively, the two Lamé constants and the bulk modulus in the natural configuration at the reference temperature Θ_0, $J = \det \mathbf{F}$, α_0 is the thermal expansion coefficient and c is the heat capacity at constant strain, assumed to be independent of the temperature Θ.

Starting from the current configuration, the propagation of small-amplitude thermo-mechanical disturbances in the plane 1-2 is analysed. With u_i $(i = 1, 2)$ and $\theta = \delta\Theta$ denoting the incremental displacement and temperature fields, respectively, the governing equations are:

i) the incremental equations of motion in the absence of body forces

$$\Sigma_{ij,j} = \rho\ddot{u}_i, \tag{5}$$

where Σ_{ij} is the incremental first Piola-Kirchhoff stress updated to the current configuration, and ρ is the mass density. A superposed dot means material time derivative;

ii) the local conservation of energy in the absence of heat sources

$$\bar{c}\,\dot{\theta} = -q_{k,k} + \Theta M_{ij}(\dot{u}_{i,j} + \dot{u}_{j,i})/2, \tag{6}$$

where $\bar{c} = c/J$, M_{ij} is the stress-temperature tensor, q_i is the heat flux, which can be expressed in terms of θ through the Fourier law

$$q_i = -K\theta_{,i},$$

where K is the thermal conductivity;

iii) the mechanical boundary conditions under incremental plane strain constraint in the transverse direction $(u_3 = 0,\ \Sigma_{13} = \Sigma_{31} = \Sigma_{23} = \Sigma_{32} = 0)$

- at a boundary subject to dead load:

$$\Sigma_{22} = 0, \quad \Sigma_{12} = 0; \tag{7}$$

- at an interface between two layers denoted by $^+$ and $^-$:

$$\Sigma_{22}^+ = \Sigma_{22}^-, \quad \Sigma_{12}^+ = \Sigma_{12}^-, \quad u_i^+ = u_i^-; \tag{8}$$

iv) the thermal boundary conditions

- at any external boundary:

$$\text{either adiabatic } \frac{\partial \theta}{\partial x_2} = 0, \text{ or prescribed temperature } \theta = \bar{\theta}; \tag{9}$$

- at an interface between two layers denoted by $^+$ and $^-$:

$$\theta^+ = \theta^-, \quad K^+ \frac{\partial \theta^+}{\partial x_2} = K^- \frac{\partial \theta^-}{\partial x_2}. \tag{10}$$

The thermoelastic constitutive equations relating the incremental displacement and temperature fields to the incremental stress may be written as

$$\Sigma_{ij} = C^{\Theta}_{ijkl} u_{k,l} + M_{ij} \theta, \tag{11}$$

where C^{Θ}_{ijkl} is the isothermal elastic tensor.

For an isotropic material the non-null components of C^{Θ}_{ijkl} are [13]

$$\begin{aligned} JC^{\Theta}_{iijj} &= \lambda_i \lambda_j \psi_{,ij}, \\ JC^{\Theta}_{ijij} &= \lambda_j^2 \frac{\lambda_j \psi_{,j} - \lambda_i \psi_{,i}}{\lambda_j^2 - \lambda_i^2}, \quad i \neq j, \ \lambda_i \neq \lambda_j, \\ JC^{\Theta}_{ijji} &= JC^{\Theta}_{ijij} - \lambda_j \psi_{,j}, \qquad i \neq j, \end{aligned} \tag{12}$$

so that for the free energy (4) and a plane-strain condition ($\lambda_3 = 1$) the in-plane, non-vanishing components are

$$C_1 = C^{\Theta}_{1111} = \mu_0 \frac{\Theta}{\Theta_0} \frac{1 + \lambda_1^2}{J} + \lambda_0 \frac{\Theta}{\Theta_0} J, \tag{13}$$

$$C_2 = C^{\Theta}_{2222} = \mu_0 \frac{\Theta}{\Theta_0} \frac{1 + \lambda_2^2}{J} + \lambda_0 \frac{\Theta}{\Theta_0} J, \tag{14}$$

$$C_3 = C^{\Theta}_{1122} = C^{\Theta}_{2211} = \lambda_0 \frac{\Theta}{\Theta_0} (2J - 1) - 3\alpha_0 \kappa_0 (\Theta - \Theta_0), \tag{15}$$

$$C_4 = C^{\Theta}_{1221} = C^{\Theta}_{2112} = \mu_0 \frac{\Theta}{\Theta_0} \frac{1}{J} - \lambda_0 \frac{\Theta}{\Theta_0} (J - 1) + 3\alpha_0 \kappa_0 (\Theta - \Theta_0), \tag{16}$$

$$C_5 = C^{\Theta}_{1212} = \mu_0 \frac{\Theta}{\Theta_0} \frac{\lambda_2^2}{J}, \qquad C_6 = C^{\Theta}_{2121} = \mu_0 \frac{\Theta}{\Theta_0} \frac{\lambda_1^2}{J}. \tag{17}$$

The stress-temperature tensor M_{ij} is given by

$$M_{ij} = \frac{\partial T_{ij}(F_{kl}, \Theta)}{\partial \Theta}, \tag{18}$$

where T_{ij} is the Cauchy stress tensor. In the Eulerian reference system, T_{ij} is diagonal with principal components $T_i = J^{-1} \lambda_i \psi_{,i}$, so that M_{ij} has also a diagonal form with components

$$M_i = \frac{\partial T_i}{\partial \Theta} = J^{-1} \lambda_i \frac{\partial^2 \psi}{\partial \lambda_i \partial \Theta}, \tag{19}$$

namely

$$M_1 = \mu_0 \, \frac{\lambda_1^2 - 1}{J} \frac{1}{\Theta_0} + \lambda_0 (J-1) \frac{1}{\Theta_0} - 3\alpha_0 \kappa_0, \tag{20}$$

$$M_2 = \mu_0 \, \frac{\lambda_2^2 - 1}{J} \frac{1}{\Theta_0} + \lambda_0 (J-1) \frac{1}{\Theta_0} - 3\alpha_0 \kappa_0. \tag{21}$$

It should be noted that in the case when the pre-strain is null, $\lambda_1 = \lambda_2 = \lambda_3 = 1$, the constitutive equation (11) reduces to

$$\delta T_{ij} = (\tilde{\lambda}\, u_{k,k} - 3\alpha_0 \kappa_0 \, \delta\Theta)\delta_{ij} + \tilde{\mu}(u_{i,j} + u_{j,i}), \tag{22}$$

where δT_{ij} denotes increment in the Cauchy stress and

$$\tilde{\lambda} = \lambda_0 \frac{\Theta}{\Theta_0} - 3\alpha_0 \kappa_0 (\Theta - \Theta_0), \quad \tilde{\mu} = \mu_0 \frac{\Theta}{\Theta_0} + \frac{3}{2}\alpha_0 \kappa_0 (\Theta - \Theta_0). \tag{23}$$

The moduli $\tilde{\lambda}$ and $\tilde{\mu}$ are independent of the incremental quantities, so that the form (22) of the constitutive law represents the well-known equations of infinitesimal thermoelasticity [14].

3. THERMOELASTIC WAVE PROPAGATION

In [15] it is shown that, for the generic layer, the displacement and temperature fields associated with a sinusoidal small-amplitude thermoelastic travelling wave propagating along direction 1 are characterized by the following expressions

$$\begin{aligned}\{u_1,\, u_2,\, \theta\} = \sum_{j=1}^{3} \; & \big[\{1, f(s_j), g(s_j)\} A_j e^{s_j k x_2} \\ & + \{1, -f(s_j), g(s_j)\} A_{j+3} e^{-s_j k x_2}\big] \, e^{ik(x_1 - vt)},\end{aligned} \tag{24}$$

where k and v are, respectively, the wavenumber and the speed of propagation of the wave, both complex numbers, but in such a way that the frequency, kv, be real. A_j $(j = 1, ..., 6)$ are unknown constants, whereas the six complex roots $s_1 = -s_4$, $s_2 = -s_5$, and $s_3 = -s_6$ are the solutions of the characteristic equation

$$\begin{aligned}&\{\bar{C}_5 s^2 - (1 + \bar{M}_1^2 \Xi) + \rho v^2 / C_1\}\{(\bar{C}_2 + \bar{M}_2^2 \Xi) s^2 - \bar{C}_6 + \rho v^2 / C_1\} \\ &+ \{(\bar{C}_3 + \bar{C}_4) + \bar{M}_1 \bar{M}_2 \Xi\}^2 s^2 \\ &+ i(1 - s^2) Q C_1 / (\rho v^2) \{(\bar{C}_5 s^2 - 1 + \rho v^2 / C_1)(\bar{C}_2 s^2 - \bar{C}_6 + \rho v^2 / C_1) + (\bar{C}_3 + \bar{C}_4)^2 s^2\},\end{aligned} \tag{25}$$

obtained from (5), where $\bar{C}_j = C_j / C_1$, $\bar{M}_j = M_j / \bar{c}$, $\Xi = \Theta \bar{c} / C_1$ and $Q = kvK\rho / (C_1 \bar{c})$.

The functions $f(s_j)$ and $g(s_j)$ are

$$f(s_j) = -s_j \frac{i\bar{M}_2(\bar{C}_5 s_j^2 + \rho v^2/C_1 - 1) + i\bar{M}_1(\bar{C}_3 + \bar{C}_4)}{\bar{M}_1(\bar{C}_2 s_j^2 + \rho v^2/C_1 - \bar{C}_6) - \bar{M}_2 s_j^2(\bar{C}_3 + \bar{C}_4)}, \tag{26}$$

and

$$g(s_j) = iQ\frac{C_1^2}{\rho v K} \frac{(\bar{C}_3 + \bar{C}_4)^2 s_j^2 + (\bar{C}_5 s_j^2 + \rho v^2/C_1 - 1)(\bar{C}_2 s_j^2 + \rho v^2/C_1 - \bar{C}_6)}{\bar{M}_1(\bar{C}_2 s_j^2 + \rho v^2/C_1 - \bar{C}_6) - \bar{M}_2 s_j^2(\bar{C}_3 + \bar{C}_4)}. \tag{27}$$

The constants A_j $(j = 1, ..., 6)$, the coefficients s_j $(j = 1, ..., 6)$ and the functions $f(s_j)$, $g(s_j)$ are, in general, different from layer to layer.

Through (24) and (11) the boundary conditions (7)-(10) must be imposed, and an eigenvalue problem for the complex propagation velocity v is obtained, which can be solved numerically. In order to sketch the adopted numerical procedure, a matrix notation is employed. For the h-th layer

$$[u_1 \;\; u_2 \;\; \theta \;\; \Sigma_{22} \;\; \Sigma_{12} \;\; \partial\theta/\partial x_2]^T = \mathsf{Y}(x_2)\mathsf{A}e^{ik(x_1 - vt)}, \tag{28}$$

where $\mathsf{A} = [A_1 \; A_2 \; A_3 \; A_4 \; A_5 \; A_6]^T$ is the vector of the arbitrary constants defined in (24), and the matrix $\mathsf{Y}(x_2)$ can be factorized in $\mathsf{Y}(x_2) = \mathsf{HE}(x_2)$, where

$$\mathsf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ f(s_1) & f(s_2) & f(s_3) & -f(s_1) & -f(s_2) & -f(s_3) \\ g(s_1) & g(s_2) & g(s_3) & g(s_1) & g(s_2) & g(s_3) \\ C_3 ik + C_2 k s_1 f(s_1) + M_2 g(s_1) & \dots & \dots & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \\ C_4 ik f(s_1) + C_5 k s_1 & \dots & \dots & -\mathsf{H}_{51} & -\mathsf{H}_{52} & -\mathsf{H}_{53} \\ k s_1 g(s_1) & \dots & \dots & -\mathsf{H}_{61} & -\mathsf{H}_{62} & -\mathsf{H}_{63} \end{bmatrix} \tag{29}$$

and $\mathsf{E}(x_2) = \text{diag}(e^{s_1 k x_2}, e^{s_2 k x_2}, e^{s_3 k x_2}, e^{-s_1 k x_2}, e^{-s_2 k x_2}, e^{-s_3 k x_2})$. Now, the three terms of $\mathsf{E}(x_2)$ which have positive real part of the exponent may trigger numerical instability due to the exponential function. To avoid this problem we further decompose $\mathsf{E}(x_2)$ in $\mathsf{E}(x_2) = \mathsf{E}^*(x_2)\mathsf{A}^*$ [16], where

$$\begin{aligned} \mathsf{E}^*(x_2) = \text{diag} \; (\; & e^{s_1 k(x_2 - x_2^{lh})}, e^{s_2 k(x_2 - x_2^{lh})}, e^{s_3 k(x_2 - x_2^{lh})}, \\ & e^{-s_1 k(x_2 - x_2^{uh})}, e^{-s_2 k(x_2 - x_2^{uh})}, e^{-s_3 k(x_2 - x_2^{uh})}), \end{aligned} \tag{30}$$

and $\mathsf{A}^* = \text{diag}(e^{s_1 k x_2^{lh}}, e^{s_2 k x_2^{lh}}, e^{s_3 k x_2^{lh}}, e^{-s_1 k x_2^{uh}}, e^{-s_2 k x_2^{uh}}, e^{-s_3 k x_2^{uh}})$ is a matrix of constants. The real parts of the exponents in (30) are always less or equal zero when the boundary conditions are imposed. The expression (28) can be written as

$$[u_1 \;\; u_2 \;\; \theta \;\; \Sigma_{22} \;\; \Sigma_{12} \;\; \partial\theta/\partial x_2]^T = \mathsf{Y}^*(x_2)\mathsf{A}^*\mathsf{A}e^{ik(x_1 - vt)}, \tag{31}$$

where vector $\mathsf{A}^*\mathsf{A}$ collects a new set of constants and the matrix $\mathsf{Y}^*(x_2) = \mathsf{HE}^*(x_2)$ has no longer the shortcoming associated with positive exponents in E^* which are responsible of bad conditioning of the problem.

As an example, solving the eigenvalue problem for a two-layer structure (see Fig. 2) means to impose vanishing determinant of the following 12 x 12 matrix:

$$\left[\begin{array}{c|c} \mathsf{Y}^{*1}_{4-6,j}(0) & 0 \\ \hline \mathsf{Y}^{*1}(h_1) & -\mathsf{Y}^{*2}(h_2) \\ \hline 0 & \mathsf{Y}^{*2}_{4-6,j}(h_1+h_2) \end{array}\right], \quad (j=1,...,6). \tag{32}$$

Figure 2: Two-layer structure.

4. RESULTS

Results relative to a two-layer structure are shown. Uniaxial tensile and compressive stress states have been analysed for simplicity, in globally adiabatic conditions, eqn. $(9)_1$. In particular, a longitudinal stretch λ_1 and a constant temperature Θ have been imposed uniformly for the whole structure. The stretch in the transversal direction λ_2 has been calculated by imposing the vanishing of the transversal stress, $T_2 = J^{-1}\lambda_2\psi_{,2} = 0$, thus obtaining a relationship determining λ_2 as a function of λ_1 and Θ. We have fixed the following values of parameters, $\alpha_0 = 10^{-4}$ K^{-1}, $\Theta_0 = 293\,$K, $c = 1.8 \cdot 10^6\,$N/(m^{2}K), $K = 0.2\,$W/m K, and we have employed the Poisson ratio ν_0 defined in the unstressed configuration and at $\Theta = \Theta_0$, which is related to the elastic constants λ_0 and μ_0 through the relationships $\nu_0 = \lambda_0/[2(\lambda_0+\mu_0)]$, $\lambda_0 = 2\,\nu_0\mu_0/(1-2\,\nu_0)$.

In the examples we explore the condition of assigned wavenumber k, which is consequently taken to be real. Obviously, the frequency $\omega = kv$ turns out to be complex. This is related to the well-known fact that waves in thermoelastic materials are always dispersive, even in the cases of propagation of body and Rayleigh disturbances [17].

Two layers of equal current thickness h, labelled with superscripts $-$ and $+$, are considered. The layers differ only for the elastic stiffness μ_0, while all other parameters have been kept uniform, included the Poisson ratio, $\nu_0 = 0.49$, the thermal expansion coefficient, the conductivity and the heat capacity. In all figures, the phase velocity Re(v) is multiplied by $\sqrt{\rho/\mu_0^-}$ and plotted versus the nondimensional wave number $kh/2$. Fig. 3 pertains to a compressive uniaxial

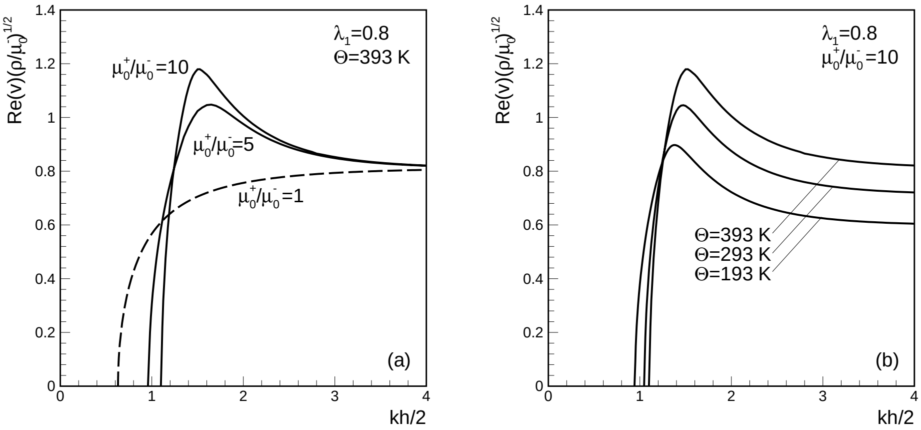

Figure 3: Dispersion diagrams for a two-layer structure, with $\lambda_1 = 0.8$, corresponding to a compressive pre-stress. The thickness of both layers is equal to h.

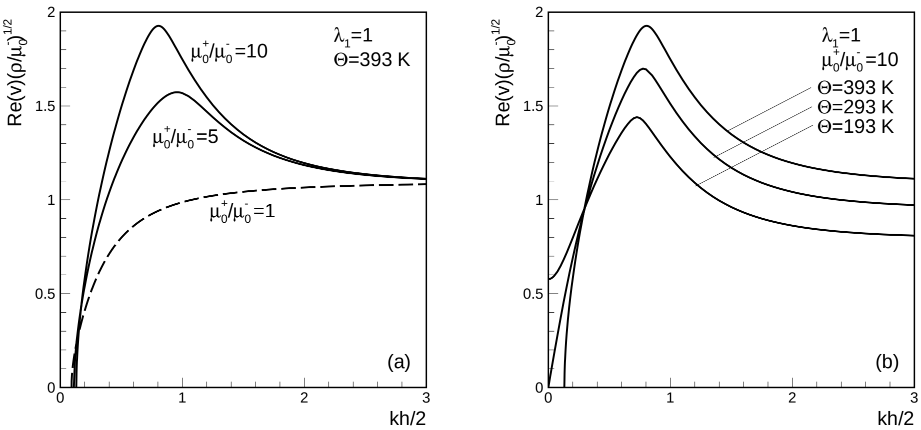

Figure 4: Dispersion diagrams for a two-layer structure, with $\lambda_1 = 1$, so that the pre-stress depends only on temperature. The thickness of both layers is equal to h.

pre-stress (for all investigated values of temperature), with a longitudinal stretch $\lambda_1 = 0.8$, whereas the pre-stress is tensile in Fig. 5, with $\lambda_1 = 1.3$.

The pre-strain is null in Fig. 4, $\lambda_1 = 1$, so that the pre-stress only depends on temperature. Three different values of ratios between elastic moduli of the two layers, $\mu_0^+/\mu_0^- = \{1,\ 5,\ 10\}$, are considered in parts (a) for $\Theta = 393$ K, whereas $\mu_0^+/\mu_0^- = 10$ in parts (b), where temperature is varied, $\Theta = \{193,\ 293,\ 393\}$ K. Obviously, when the ratio between stiffnesses equals unity, $\mu_0^+/\mu_0^- = 1$, the two layers behave as a single layer of thickness $2h$ (dashed curve).

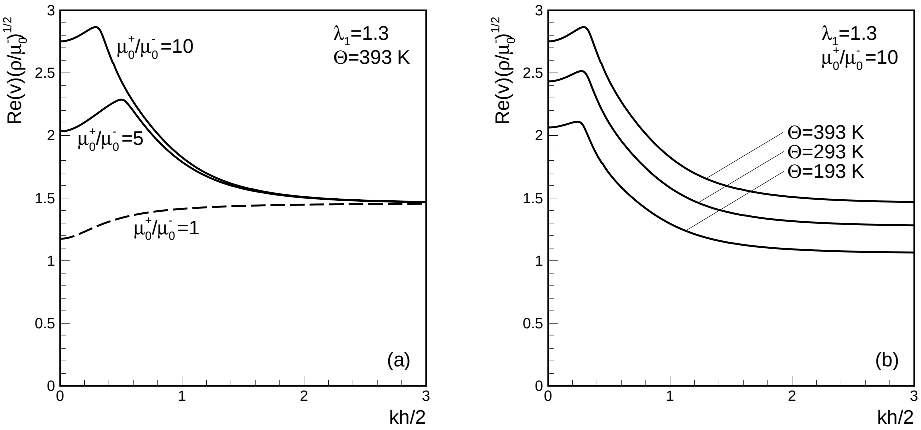

Figure 5: Dispersion diagrams for a two-layer structure, with $\lambda_1 = 1.3$, corresponding to a tensile pre-stress. The thickness of both layers is equal to h.

Points in the graphs corresponding to a null velocity represent bifurcation conditions in a beam-like buckling mode. It is clear for $\lambda_1 = 1$, i.e. from Fig. 4(b), that an increase in temperature, which induces a compressive pre-stress, promotes bifurcation, whereas the occurrence of buckling may be eliminated by a decrease in temperature. This behaviour is also obviously influenced by the value of current stretch, which strongly modifies the state of pre-stress.

It appears clearly that temperature has a quantitative effect on the graphs, but not qualitative, whereas changing the stiffness ratio of the structure yields a strong, qualitative effect. An interesting feature of the results is that when the stiffness ratio increases, the curves evidence a peak so that propagation velocities initially increase at increasing wavenumber, but then a maximum is reached and speeds become decreasing functions of wavenumber.

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