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Edited by
R. CHAMBON & J. DESRUES
Laboratoire Sols, Solides, Structures (3S), UJF – INPG – CNRS, Grenoble, France
I. VARDOULAKIS
National Technical University, Athens, Greece

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A dynamical interpretation of flutter instability

Une interprétation dynamique de l'instabilité de flottement

D. Bigoni
Istituto di Scienza delle Costruzioni, University of Bologna, Italy

J. R. Willis
School of Mathematical Sciences, University of Bath, UK

ABSTRACT: The differential equations governing the development of a small disturbance in material initially stressed to the flutter condition are shown to admit no solution, except for special sets of initial conditions. A viscous regularization, which modifies the dynamic but preserves the static response of the material, is introduced, which permits the solution of the incremental initial value problem for all initial data. The results provide the first physical interpretation of the flutter instability proposed by Rice (1976). In particular, flutter corresponds to an oscillating motion of the material particles, which blows up with time. This behaviour is similar to what is observed in structural systems subjected to follower loads.

On montre que les équations différentielles qui gouvernent le développement d'une petite perturbation dans un matériau soumis à un état de contrainte qui vérifie la condition de flottement n'admettent pas de solution, sauf pour des conditions initiales particulières. Une régularisation viscoplastique est introduite, qui modifie la réponse dynamique mais préserve la réponse statique; ceci permet d'obtenir une solution du problème de Cauchy incrémental pour toutes les données initiales. Le résultat fournit la première interprétation physique de l'instabilité de flottement proposée par Rice (1976). En particulier, le flottement correspond à un mouvement oscillatoire des particules, avec une solution explosive dans le temps. Ce comportement est similaire à celui qu'on observe dans les structures soumises à des charges suiveuses.

1 INTRODUCTION

Flutter instabilities occurring in structural elements subjected to follower loads have been known from the early works of Nikolai (1928), Pflüger (1950), Beck (1952) and Ziegler (1953, 1956) and have been thoroughly studied in succeeding years (Bolotin 1963, Leipholz 1964, Herrmann and Jong 1965, Como 1966, Augusti 1966, Nemat-Nasser and Herrmann 1966a, 1966b, Prasad and Herrmann 1969, Dubey and Leipholz 1975, Alliney and Tralli 1984, Laudiero et al. 1991). In the case of a structural system, flutter instability consists of a vibrational motion of increasing amplitude, when adjacent configurations of static equilibrium for the system are absent. This circumstance occurs when the eigenvalue problem governing the vibration frequencies of the system admits complex eigenvalues. In particular, the condition for the onset of flutter is given by the coalescence of two eigenvalues. For continuous media, Rice (1976) considered the eigenvalues of the acoustic tensor in the context of studying localization. He termed the situation where two real eigenvalues coalesce and then move into the complex plane, that of flutter instability, by analogy with terminology for structures. In contrast to the mechanics of structures, the flutter instability so defined for continuous media is not understood in dynamical terms. In fact, all work to date has concerned evaluations of the constitutive parameters for the occurrence of flutter (Loret et al. 1990, Loret and Harireche 1991, An and Schaeffer 1992, Loret 1992, Bigoni and Zaccaria 1992, 1994, Bigoni 1994).

A brief review of the papers on flutter in continuous media reveals that flutter is much more frequent than one might expect. In fact, flutter was detected in the case of mixture theories of plasticity (Loret and Harireche 1991) and for finite theories of plasticity in the presence of hypoelasticity with asymmetric constitutive law (An and Schaeffer 1992, Bigoni 1994). Moreover, in the usual theories of infinitesimal elastoplasticity, the *onset* of flutter, i.e. the coalescence of two eigenvalues of the acoustic tensor, is always possible, even for associative flow-law. Therefore, a generic perturbation can always induce flutter. For instance, in the case of flow laws obeying deviatoric associativity, a perturbation in the direction of the plastic flow, non-coaxial with the yield function gradient, is sufficient to yield flutter (Loret 1992).

All of the works quoted above refer to the algebraic condition of occurrence of flutter in a continuous medium, without any exploration of the physical meaning of the criterion. The purpose of the present work is to show with a simple example how flutter instability may on one hand be related to the integrability of differential equations governing the dynamic modion of a body, and on the other hand to a particular type of dynamic instability. In the example that we will consider, it is important to observe that flutter instability occurs even if the constitutive operator is positive definite, and therefore second order work positiveness and strong ellipticity are verified. This circumstance suggests that the classical definitions of material stability should be extended to cover the possibility of flutter.

The problem which is addressed in this work corresponds to a medium which is stable (i.e. has acoustic tensor with real and positive eigenvalues) up to some level of stress but, at some critical level, two eigenvalues coalesce and thereafter turn into the complex plane. It is envisaged that the body is at rest, in (unstable) equilibrium, in a state of uniform stress just greater than that associated with coalescence. For the sake of this first investigation, a particular constitutive equation has been selected, which corresponds to a non-symmetric linear constitutive operator, with two complex conjugate eigenvalues. The acoustic tensor, corresponding to the particular constitutive law, has two complex conjugate eigenvalues too. It should be noted that the assumed constitutive law is linear but it may correspond to the loading branch of an elastoplastic constitutive operator. For this problem, it will be shown that the dynamic equations of motion do not possess solutions for arbitrary initial conditions — that is, that the incremental dynamic initial value problem is ill-posed. Therefore, a viscous regularization is introduced by assuming a particular viscous response of the material under shear. The response of the medium to quasi-static deformations is unaltered. The solution shows that, after loading by a small impulse, two symmetric waves travel in opposite directions in the body. There is a wave front across which the displacements suffer a finite jump. Contrary to the well-known elastic symmetric solution (see, e.g. Graff 1975), the displacements are not constant behind the wave front. Therefore, the material particles experience an oscillation in time, after the passage of the wave front. The oscillation grows exponentially with time, remaining finite for every finite value of time. Therefore, we propose to interpret this behaviour, which is similar to the motion experienced by structural elements under follower loads, as a physical consequence of flutter. It should be mentioned that the conclusions presented herein have some similarities with those of Sandler and Rubin (1987). However, the instabilities discussed in that work related to the loading/unloading behaviour, whereas here it is demonstrated that even incremental loading presents difficulties associated with ill-posedness, unless the governing equations are modified.

2 NON-EXISTENCE OF POST-FLUTTER SOLUTIONS

We consider a medium, uniformly stressed into the flutter regime. Its plane-strain response to small perturbations which generate disturbances depending on x_1 and t only is described by the constitutive relation:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 1 & \varepsilon \\ -\varepsilon & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{,1}, \tag{2.1}$$

where σ_{ij} and u_i are the stress and displacement components, respectively. ε is a (positive) scalar parameter which provides the asymmetry of the constitutive operator. It should be noted that the constitutive operator has two complex conjugate eigenvalues, both with positive real part. The equations of motion of a wave in such a material are (ρ is the mass density, t the time variable and x the space variable, which coincides with the direction 1):

$$\frac{1}{\rho} \begin{bmatrix} 1 & \varepsilon \\ -\varepsilon & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{,xx} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{,tt}. \tag{2.2}$$

Therefore, the constitutive operator (2.1) coincides with the acoustic tensor. The equations of motion (2.2) are strongly elliptic, and thus no characteristic solutions are possible.
Now solutions are sought in the following form:

$$\mathbf{u} = \mathbf{f}[x + c(1 + i\varepsilon)^{1/2} t], \tag{2.3}$$

and therefore

$$\mathbf{u}_{,xx} = \mathbf{f}'' \text{ and } \mathbf{u}_{,tt} = c^2(1 + i\varepsilon)\mathbf{f}''. \tag{2.4}$$

It can be easily verified that

$$\mathbf{f} = \begin{bmatrix} 1 \\ i \end{bmatrix} g[x + c(1 + i\varepsilon)^{1/2} t], \tag{2.5}$$

is a solution of (2.2) for any function g. Another solution is

$$\mathbf{f} = \begin{bmatrix} 1 \\ i \end{bmatrix} h[x - c(1+i\varepsilon)^{1/2}t], \tag{2.6}$$

and the complex conjugate functions are solutions as well. Therefore the general solution is:

$$\mathbf{u} = \mathcal{R}e \begin{bmatrix} 1 \\ i \end{bmatrix} \times$$
$$\times \left\{ g[x + c(1+i\varepsilon)^{1/2}t] + \overline{h}[x - c(1+i\varepsilon)^{1/2}t] \right\}. \tag{2.7}$$

Let us analyze the initial value problem

$$\mathbf{u}(x,0) = 0, \ \mathbf{u}_{,t}(x,0) = \mathbf{v}(x). \tag{2.8}$$

The initial conditions (2.8) imply

$$g(x) = -\overline{h}(x), \tag{2.9}$$

$$\mathbf{u}_{,t}(x,0) = +2\mathcal{R}e \begin{bmatrix} 1 \\ i \end{bmatrix} \left\{ c(1+i\varepsilon)^{1/2} g'(x) \right\}, \tag{2.10}$$

i.e.

$$\mathbf{u}_{,t}(x,0) =$$
$$+ \begin{bmatrix} c[(1+i\varepsilon)^{1/2}g'(x) + (1-i\varepsilon)^{1/2}\overline{g}'(x)] \\ ci[(1+i\varepsilon)^{1/2}g'(x) - (1-i\varepsilon)^{1/2}\overline{g}'(x)] \end{bmatrix}. \tag{2.11}$$

If, in particular,

$$\mathbf{u}_{,t}(x,0) = \begin{bmatrix} v_1(x) \\ 0 \end{bmatrix}, \tag{2.12}$$

one obtains

$$(1-i\varepsilon)^{1/2}\overline{g}'(x) = (1+i\varepsilon)^{1/2}g'(x), \tag{2.13}$$

$$v_1(x,0) = +2c(1+i\varepsilon)^{1/2}g'(x), \tag{2.14}$$

which are mutually inconsistent unless v_1 is an analytic function of x, real when x is real. This is not the case, for example when

$$v(x) = \mathbf{V}\delta(x). \tag{2.15}$$

To verify this claim, let us assume $\rho = 1$, for simplicity, and re-write the differential problem (2.2), with initial conditions (2.8), with $v(x)$ given by (2.15),

$$\mathbf{C}\mathbf{u}_{,xx} + \mathbf{V}\delta(x)\delta(t) = \mathbf{u}_{,tt}, \tag{2.16}$$

where C is the constitutive tensor defined in (2.1). Taking the Fourier transform on the time variable, one gets (a superscript $\hat{}$ denotes the Fourier transform of a function, a superscript $-$ denotes the complex conjugate of a number):

$$\mathbf{C}\hat{\mathbf{u}}_{,xx} + \omega^2\hat{\mathbf{u}} + \mathbf{V}\delta(x) = 0, \tag{2.17}$$

where

$$\hat{u}(x,\omega) = \int_{-\infty}^{\infty} u(x,t)e^{i\omega t}dt. \tag{2.18}$$

By introducing the change of variables

$$\hat{\mathbf{u}} = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} \hat{\mathbf{w}}, \tag{2.19}$$

the differential problem (2.17) becomes

$$\begin{cases} \hat{w}_{1,xx} + \Omega_+^2\hat{w}_1 + \frac{1}{2}\{i,1\}\{V\}\frac{i}{i-\varepsilon}\delta(x) = 0 \\ \hat{w}_{2,xx} + \Omega_-^2\hat{w}_2 + \frac{1}{2}\{-i,1\}\{V\}\frac{i}{i+\varepsilon}\delta(x) = 0 \end{cases}, \tag{2.20}$$

where $\{V\}$ represents the column vector corresponding to V and

$$\Omega_+^2 = \omega^2\frac{i}{i-\varepsilon} \ \text{ and } \ \Omega_-^2 = \omega^2\frac{i}{i+\varepsilon}. \tag{2.21}$$

Solutions of (2.20) are selected in the form:

$$\begin{cases} \hat{w}_1 = \frac{i}{2\Omega_+}\frac{1}{2}\{i,1\}\{V\}\frac{i}{i-\varepsilon}e^{i\Omega_+|x|} \\ \hat{w}_2 = \frac{i}{2\Omega_-}\frac{1}{2}\{-i,1\}\{V\}\frac{i}{i+\varepsilon}e^{i\Omega_-|x|} \end{cases}. \tag{2.22}$$

The exact choices of the branches for Ω_+, Ω_- are imposed by the requirement of causality, which implies that $\hat{w}_1, \hat{w}_2$ must be analytic in the lower half of the complex ω-plane. Thus, $\Omega_+ = \omega(1+i\varepsilon)^{-1/2}, \Omega_- = \omega(1-i\varepsilon)^{-1/2}$ and the square roots have positive real parts. It follows that

$$\begin{cases} u_1 = \{\mathcal{R}e(I_1), \mathcal{I}m(I_1)\}\{V\} \\ u_2 = \{-\mathcal{I}m(I_1), \mathcal{R}e(I_1)\}\{V\} \end{cases}, \tag{2.23}$$

where

$$I_1(x,t) =$$
$$\frac{i}{8\pi}\{i,1\}\{V\}\left[\frac{1}{1+i\varepsilon}\right]^{1/2}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{i\omega|x|\left[\frac{1}{1+i\varepsilon}\right]^{1/2}}e^{-i\omega t}d\omega. \tag{2.24}$$

It is easy now to show that I_1 diverges. In fact,

$$\frac{\partial I_1}{\partial t} = -i\left[\frac{1}{i|x|\left[\frac{1}{1+i\varepsilon}\right]^{1/2} - t}e^{i\omega|x|\left[\frac{1}{1+i\varepsilon}\right]^{1/2}}e^{-i\omega t}\right]_{-\infty}^{\infty}, \tag{2.25}$$

in which $\left[\frac{1}{1+i\varepsilon}\right]^{1/2} = \left[\frac{1-i\varepsilon}{1+\varepsilon^2}\right]^{1/2}$ has negative imaginary part, and therefore (2.25) grows when $\omega \to \infty$.

In conclusion of this Section, we stress that we are unable to solve the dynamic problem (2.2) with general initial impulse conditions (2.8). This fact shows clearly that *in the case of flutter the equations of motion governing small perturbations may have no solution.* The implications of this finding for numerical methods are evident.

3 VISCOUS REGULARIZATION AND SOLUTION OF INITIAL VALUE PROBLEM UNDER FLUTTER CONDITIONS

In this Section the problem posed in the previous Section is resolved by introducing a viscous regularization. To this purpose, the following viscous-non-symmetric constitutive equation is assumed for the body:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \int_{-\infty}^{\infty} dt'\delta(t-t') & \frac{\varepsilon}{\tau}\int_{-\infty}^{\infty} dt' H(t-t')e^{-(t-t')/\tau} \\ -\frac{\varepsilon}{\tau}\int_{-\infty}^{\infty} dt' H(t-t')e^{-(t-t')/\tau} & \int_{-\infty}^{\infty} dt'\delta(t-t') \end{bmatrix} \times \begin{bmatrix} u_{1,1}(t') \\ u_{2,1}(t') \end{bmatrix}, \tag{3.1}$$

where $H(\cdot)$ is the Heaviside step function, t is the time, ε and τ are material parameters, σ_{ij} are the stress and u_i the displacement components. It is important to note that the scalar parameter ε is related to the non-symmetry of the constitutive equation, whereas τ is related to the viscosity of the material when subjected to shear. By introducing the constitutive matrix $[C]$ and the column vectors $\{\sigma\}$ and $\{u\}$, equation (3.1) may be written as

$$\{\sigma\} = [C]\{u\}_{,x} \tag{3.2}$$

The problem to be considered is the motion of an infinite body, governed by the constitutive equation (3.1), with density $\rho = 1$, and loaded by an impulse load occurring at $t = 0$ over all points of the plane $x = 0$. For this problem, the equation of the motion can be written as

$$[C]\{u\}_{,xx} + \{V\}\delta(x)\delta(t) = \{u\}_{,tt} \ , \tag{3.3}$$

where $\{V\}$ is the vector which specifies the initial conditions. By taking the Fourier transform of (3.3) on the time variable and using the convolution theorem, one obtains the following differential problem:

$$\begin{bmatrix} 1 & \frac{\varepsilon}{1-i\omega\tau} \\ -\frac{\varepsilon}{1-i\omega\tau} & 1 \end{bmatrix} \{\hat{u}\}_{,xx} + \omega^2\{\hat{u}\} + \{V\}\delta(x) = 0 \ . \tag{3.4}$$

By introducing the following change of variables:

$$\{\hat{u}\} = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} \{\hat{w}\} \ , \tag{3.5}$$

the differential problem (3.4) becomes:

$$\begin{cases} \hat{w}_{1,xx} + \Omega_+^2 \hat{w}_1 + \frac{1}{2}\{i,1\}\{V\}\frac{i+\omega\tau}{i+\omega\tau-\varepsilon}\delta(x) = 0 \\ \hat{w}_{2,xx} + \Omega_-^2 \hat{w}_2 + \frac{1}{2}\{-i,1\}\{V\}\frac{i+\omega\tau}{i+\omega\tau+\varepsilon}\delta(x) = 0 \end{cases} \tag{3.6}$$

where

$$\Omega_+^2 = \omega^2 \frac{i+\omega\tau}{i+\omega\tau-\varepsilon} \quad \text{and} \quad \Omega_-^2 = \omega^2 \frac{i+\omega\tau}{i+\omega\tau+\varepsilon} \ . \tag{3.7}$$

Solutions of (3.6) are selected in the form:

$$\begin{cases} \hat{w}_1 = \frac{i}{2\Omega_+}\frac{1}{2}\{i,1\}\{V\}\frac{i+\omega\tau}{i+\omega\tau-\varepsilon}\, e^{i\Omega_+|x|} \\ \hat{w}_2 = \frac{i}{2\Omega_-}\frac{1}{2}\{-i,1\}\{V\}\frac{i+\omega\tau}{i+\omega\tau+\varepsilon}\, e^{i\Omega_-|x|} \end{cases} \ . \tag{3.8}$$

The use of condition (3.5), together with the causality argument given earlier, yields the antitransform:

$$\begin{cases} u_1 = \{\mathcal{R}e(I_1), \mathcal{I}m(I_1)\}\{V\} \\ u_2 = \{-\mathcal{I}m(I_1), \mathcal{R}e(I_1)\}\{V\} \end{cases} \ , \tag{3.9}$$

where

$$I_1(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{i}{2\Omega_+}\frac{i+\omega\tau}{i+\omega\tau-\varepsilon}\, e^{i\Omega_+|x|}\, e^{-i\omega t} d\omega \ . \tag{3.10}$$

The integral (3.10) can be rewritten in the form:

$$I_1(x,t) = \frac{i}{4\pi}\int_{-\infty}^{\infty} \frac{1}{\omega}\left[\frac{i+\omega\tau}{i+\omega\tau-\varepsilon}\right]^{\frac{1}{2}} e^{i\Omega_+|x|}\, e^{-i\omega t} d\omega \ . \tag{3.11}$$

The integral I_1 has a simple pole at $\omega = 0$ and two branch points at $\omega = (\varepsilon - i)/\tau$ and $\omega = -i/\tau$ (see Fig.1). When $\omega \to \infty, \Omega_+ \sim \omega$; therefore, for $t - |x| < 0$ closing the contour in the upper half plane gives zero. Thus, a wave front does exist, corresponding to $|x|/t = 1$. For $t - |x| > 0$, closing the contour in the lower half plane yields:

$$I_1 = -2\pi i \mathcal{R}es(I_1, 0) - \frac{i}{4\pi}\oint \frac{1}{\omega}\left[\frac{i+\omega\tau}{i+\omega\tau-\varepsilon}\right]^{\frac{1}{2}} e^{i\Omega_+|x|}\, e^{-i\omega t} d\omega \ , \tag{3.12}$$

where the contour integral is to be evaluated on any closed contour enclosing the branch cut, and

$$\mathcal{R}es(I_1, 0) = \frac{i}{4\pi}\left[\frac{i}{i-\varepsilon}\right]^{\frac{1}{2}} \ . \tag{3.13}$$

The following coordinate transformation can be performed in the contour integral:

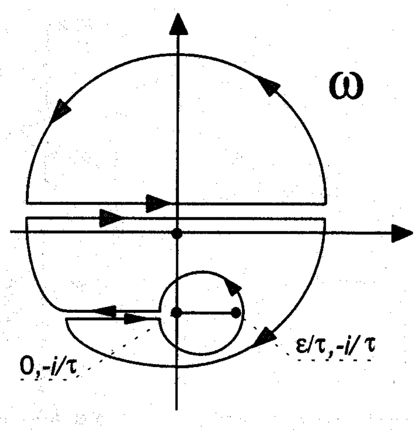
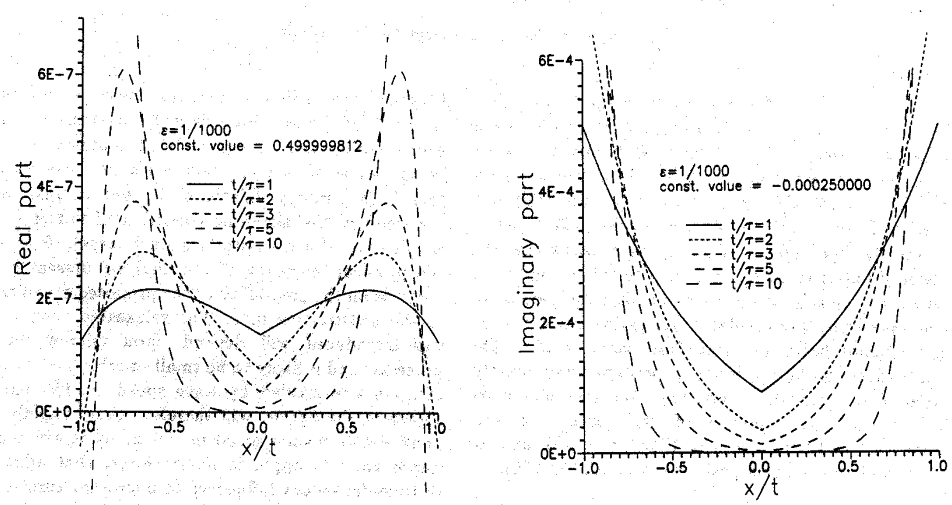


Fig 1. The complex ω-plane and the contours employed in the evaluation of I_1.



(a) (b)

Fig 2. The variation of the real part (a) and the imaginary part (b) of the integral I_1, plotted against x/t for various t/τ, when $\varepsilon = 1/1000$.

$$s = \frac{i + \omega\tau}{\varepsilon}\,, \tag{3.14}$$

therefore obtaining:

$$I_1 = \frac{1}{2}\left[\frac{i}{i-\varepsilon}\right]^{\frac{1}{2}} - \frac{i\varepsilon}{4\pi}\, e^{-t/\tau}$$

$$\oint \frac{1}{\varepsilon s - i}\left[\frac{s}{s-1}\right]^{\frac{1}{2}} e^{-i\varepsilon s t/\tau}\, e^{\frac{|x|}{t}\frac{t}{\tau}(1+i\varepsilon s)\sqrt{s/(s-1)}}\, ds\,. \tag{3.15}$$

The last integral may be evaluated numerically, e.g. on the circle $s(\vartheta) = c + re^{i\vartheta}$, where $\vartheta \in [-\pi, \pi]$.

It is worth noting that the value of the integral I_1 is given by the sum of a contribution independent of t, x and τ (which will be called "constant" in the figures) and of a contribution dependent on t/τ and $|x|/t$. Numerical values of the integral in equation (3.15) are reported in Figs. 2(a,b), 3(a,b), 4(a,b), for $0 \leq |x|/t \leq 1$ and different values of t/τ (1,2,3,5,10). Values of $\varepsilon = 1/10, 1/100, 1/1000$ are explored. In figures

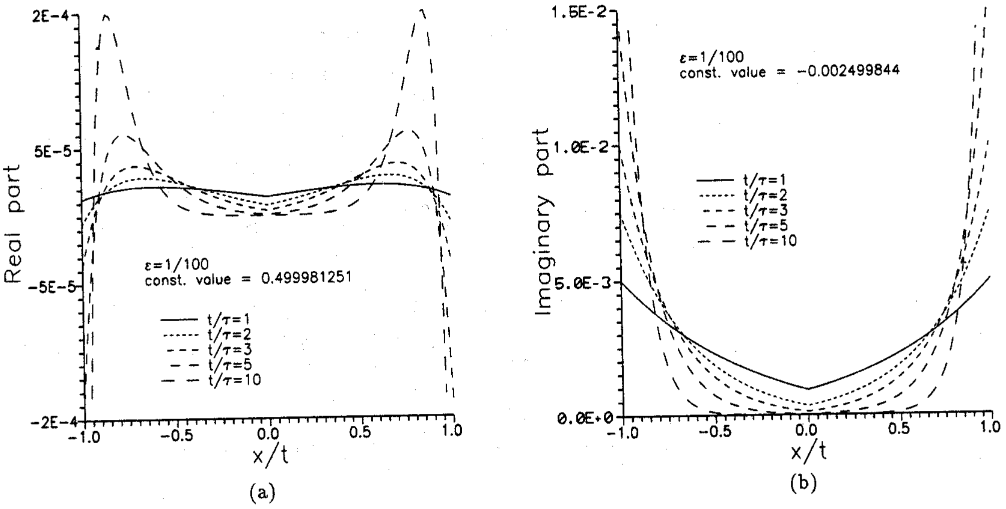


Fig 3. As for Fig. 2, except that $\varepsilon = 1/100$.

"a" the real part of the contour integral appearing in (3.15) is reported, whereas figures "b" refer to the imaginary component. From such figures it can be seen that all curves approach a well-defined value on the wave front $|x|/t = 1$. Moreover, the qualitative trends of the curves are identical for different values of ε. Finally, from figures "a", it appears clearly that the effect of flutter consists, according to this model, of an oscillating motion of the particles after the passage of the wave front. The oscillation blows up when $t/\tau \to \infty$ but remains finite for every finite value of t/τ. The same effect can be appreciated, perhaps more directly, from Fig. 5(a,b), where the real and imaginary components are reported of the contour integral appearing in (3.15), for $\varepsilon = 1/100, \tau = 100$, and for three different particle positions ($x = 10, 100, 200$).

4 CONCLUSIONS

In order to appreciate the effect of flutter, a simple problem of dynamic motion of a continuous medium has been analyzed, namely, the propagation of a small disturbance in a space of material. A first constitutive law was analyzed, for which the acoustic tensor and the constitutive tensor have two complex conjugate eigenvalues with positive real parts. In this case the dynamic equation of motion cannot be solved. For the same case, a viscous regularization has been introduced and the problem solved. The main difference with the same wave problem in the case of a (symmetric) elastic material is that, after the passage of the wave front, the material particles suffer an oscillation which blows up when time increases. However, the oscillation remains finite for every finite value of the ratio time/viscosity parameter. It is worth noting that the considered material is stable in the sense of second order work positiveness (and thus it is strongly elliptic also), but is unstable in the sense of the algebraic condition of flutter (which is calculated for quasistatic disturbances). Of course, the physical relevance of the analysis presented here depends on the credibility of the proposed modification to the constitutive law. The relaxation time τ that was introduced will depend upon micromechanical processes and is likely to be small - perhaps of the order of grain size divided by wave speed c. The blow-up shown in the figures will therefore occur rapidly, and nonlinear terms neglected in this analysis will become important. It appears, nevertheless, that admission of time-dependent influences of microstructural events will be essential if the evolution of disturbances from the "flutter" state are to be resolved.

The model presented here was selected for the purpose of illustration only; the development and analysis of more realistic models, respecting specific microstructural features, is a subject of on-going study.

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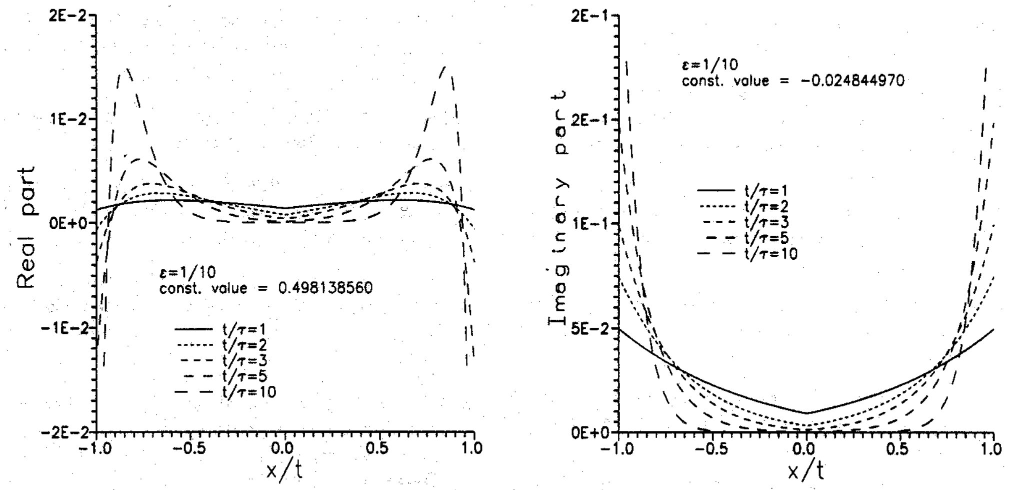
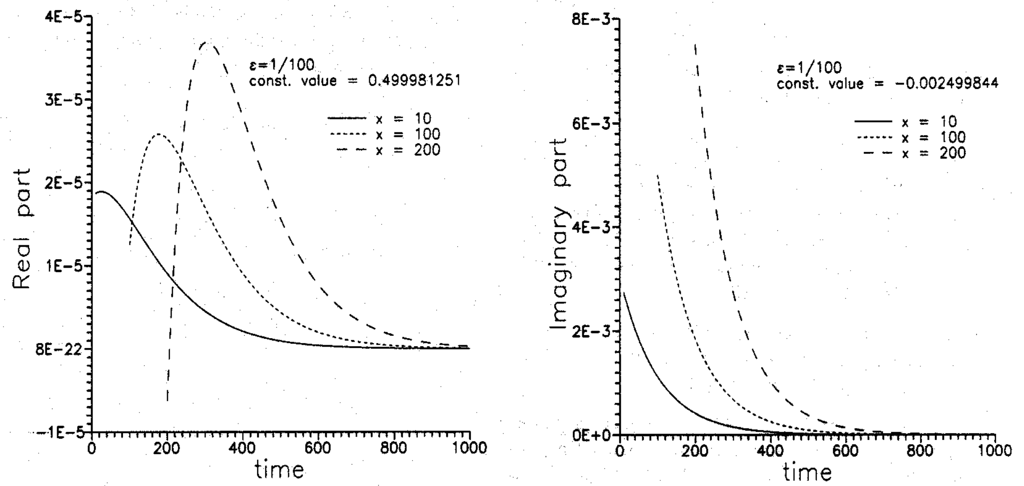


(a) (b)

Fig 4. As for Fig. 2, except that $\varepsilon = 1/10$.



(a) (b)

Fig 5. Plots of the variation of the real part (a) and the imaginary part (b) of the integral I_1 against t, for three selected positions x, showing growth of the disturbance immediately behind the wave-front, and subsequent decay.

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