The Fortran90 code **BEMdyn-incompressible** is a general boundary element software suitable to solve problems of incompressible incremental non-linear elasticity with time-harmonic external loads of prescribed circular frequency, with reference to the paper [1]. In particular :

- the Green's functions based on the Biot constitutive equations have been implemented;
- axes of orthotropy and principal directions of pre-stress coincide with the x_1 and x_2 axes;
- an uniform mesh of rectilinear elements has to be employed for the boundary discretization;
- an anti-clockwise numeration for the nodes is adopted;
- linear shape functions have been adopted for incremental displacements and tractions at the boundary;
- integrals involved in the Green's functions and in the boundary element integral equations have been evaluated by means of Gauss quadrature rules. 12 Gauss points for the Green's functions (NG) and 18 for the boundary integral equations (NB) are used.

The executable **BEMdyn-incompressible.exe** to solve two-dimensional time-harmonic problems presented in [1], is given here. To run the executable, an input file is needed (note that the input file should be located in the same directory containing the executable program). The input file can be created with any text editor (e.g. Notepad) and has to be saved without extension. It contains the geometry of the domain and the boundary conditions. The other input parameters that can be assigned by the user when the program is executing , are (Fig. 1),:

- material parameters *mu* and *mu_star*, respectively μ and μ* in [1] ;
- hydrostatic (*chi*) and deviatoric (*k*) components of pre-stress, respectively χ and k in [1];
- frequency *omega*, Ω in [1].

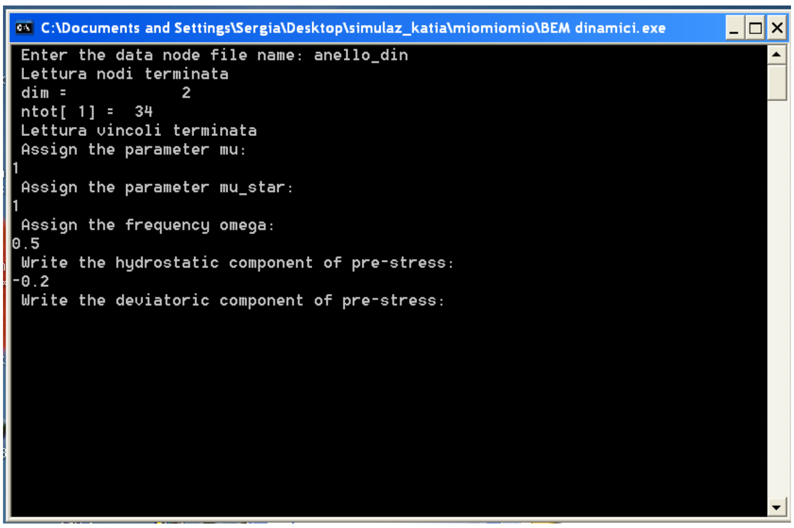

Figure 1: execution of the program with the definition of the parameters.

A directory named **result** should also be created within the directory where the executable is located. All the output files will be saved in that folder. The output files are:

1) *geom_nameinputfile.txt* where the data regarding the initial geometry are reported;

2) *spost_nameinputfile.txt* where nodal displacements and tractions are reported.

To clarify the use of the program we provide two examples, corresponding to the solution of the problems presented in sections 6.1 and 6.3 of [1]. For both of them, the input and the output files are attached in this website.

Hollow cylinder subjected to pulsating internal pressure

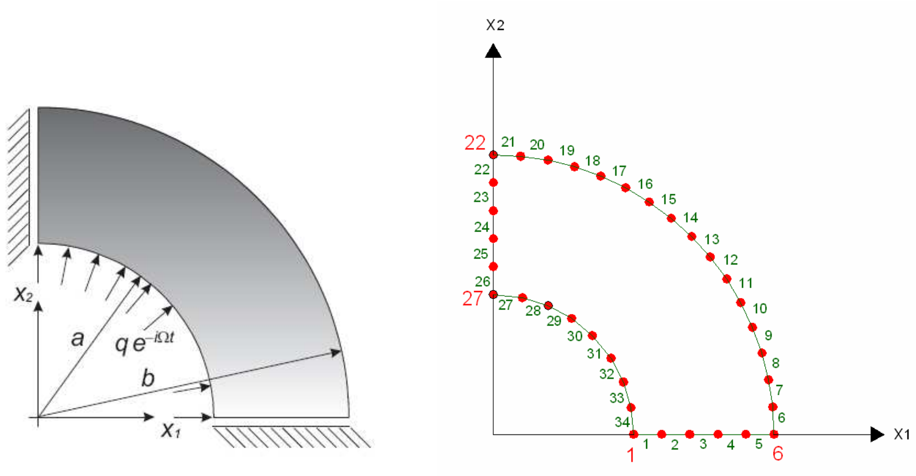

Figure 2: geometry of the problem and discretization of the boundary (boundary nodes in red colour and boundary elements in green colour).

Due to symmetry considerations only a quarter of the geometry has been considered.
Since the geometry is curvilinear, the boundary has been discretized by means of 34 linear elements of equal length (Fig. 2).

For the problem described in Fig. 1, the input file (*ring_dyn*) has the following form:

*DIM

2 →dimension of the problem (always 2 for two-dimensional problems).

*NEL

1 → number of domains (always 1 with a single domain).

*NNOD → in the follow the geometry is described.

1 34 → number of the first and of the last node of the discretization.

*NODE

1 → number of the first node.

1 1.00 0.00 → coordinates of the first node in a x_1-x_2 coordinate system.

1 → number of nodes after the previous one.

2 1.2148 0.00 → coordinates of the last node in a x_1-x_2 coordinate system.

…

and so on

…

35 1.00 0.00 → coordinates of the last node that must coincide with the first one.

0 → a zero must follow at the end.

*BOUNDARY → in the follow the boundary conditions are prescribed: displacements in the nodes and tractions on the elements.

1 → number of the domain (always 1).

1 6 0 1 0.00 0.00 → it means "from node 1 to node 6" assign displacement along x_2 and make it equal to zero".

22 27 1 0 0.00 0.00 → it means "from node 22 to node 27" assign displacement along x_1 and make it equal to zero".

0 0 0 0 0.00 0.00 → after that all nodal displacements are assigned a row of zeros must follow. In the follow tractions on the elements of the boundary are prescribed.

27 27 1 1 0.097 0.995 → it means " on element 27 assign a traction along x_1 and along x_2 which values are respectively 0.097 and 0.955".

…

and so on

…

0 0 0 0 0.00 0.00 → after that all tractions have been assigned a row of zeros must follow.

Note that where a displacement is not defined, the traction on the corresponding element is prescribed.

After that the program has been executed, two output files in the directory **result** are created. In the file *geom_ring_dyn.txt* the following data are reported:

- the number of the nodes and their coordinates x_1 and x_2;
- the number of the elements and the coordinates of their nodes;
- the number of the elements, their length and the components n_1 and n_2 of the unit outward normal;

- the assigned boundary conditions: for each boundary node, the value of real and imaginary part of displacements and for each boundary element, the value of real and imaginary part of tractions.

In the file *spost_ring_dyn.txt* the following data are reported:

- the number of boundary nodes and the value of their displacements and tractions (real and imaginary part) along x_1 and x_2, as calculated through a system of boundary integral equations.

Elastic block subject to time-harmonic antisymmetric perturbation

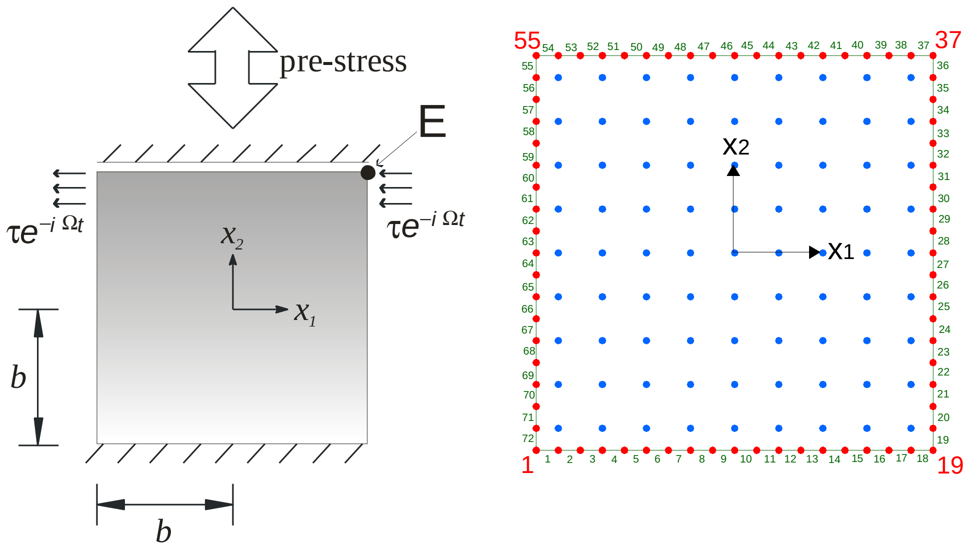

Figure 3: geometry of the problem and discretization of the boundary
(boundary nodes in red colour, internal nodes in blue colour and boundary elements in green colour).

For the problem described in Fig. 3, the input file (*square72*) has the following form:

*DIM	
2	→dimension of the problem (always 2 for two-dimensional problems).
*NEL	
1	→ number of domains (always 1 with a single domain).
*NNOD	→ in the follow the geometry is described.
1 72	→ number of the first and of the last node of the discretization.
*NODE	
1	→ number of the first node.
1 -1.00 -1.00	→ coordinates of the first node in a x_1-x_2 coordinate system.
18	→ number of nodes added.
19 1.00 -1.00	→ coordinates of the last node in a x_1-x_2 coordinate system.

18 → number of nodes added.

37 1.00 1.00 → coordinates of the last node in a x_1-x_2 coordinate system.

18 → number of nodes added.

55 -1.00 1.00 → coordinates of the last node in a x_1-x_2 coordinate system.

18 → number of nodes added.

73 -1.00 -1.00 → coordinates of the last node in a x_1-x_2 coordinate system.

0 → a zero must follow at the end.

*BOUNDARY → in the follow the boundary conditions are prescribed: displacements in the nodes and tractions on the elements.

1 → number of the domain (always 1).

1 19 1 1 0.00 0.00 → it means "from node 1 to node 19" assign displacement along x_1 and x_2 and make them equal to zero".

37 55 0 1 0.00 0.00 → it means "from node 37 to node 55" assign displacement along x_2 and make it equal to zero".

0 0 0 0 0.00 0.00 → after that all nodal displacements are assigned a row of zeros must follow. In the follow tractions on the elements of the boundary are prescribed.

19 33 1 1 0.00 0.00 → it means " from element 19 to element 33 assign null tractions along x_1 and x_2 ".

34 36 1 1 -1.00 0.00 → it means " from element 34 to element 36 assign a negative traction along x_1 and null traction along x_2 ".

55 57 1 1 -1.00 0.00 → it means " from element 55 to element 57 assign a negative traction along x_1 and null traction along x_2 ".

58 72 1 1 0.00 0.00 → it means " from element 58 to element 72 assign null tractions along x_1 and x_2 ".

0 0 0 0 0.00 0.00 → after that all tractions have been assigned a row of zeros must follow.

After that the program has been executed, two output files in the directory **result** are created. In the file *geom_ square72.txt* the following data are reported:

- the number of the nodes and their coordinates x_1 and x_2;
- the number of the elements and the coordinates of their nodes;
- the number of the elements, their length and the components n_1 and n_2 of the unit outward normal;

- the assigned boundary conditions: for each boundary node, the value of real and imaginary part of displacements and for each boundary element, the value of real and imaginary part of tractions.

In the file *spost_ square72.txt* the following data are reported:

- the number of boundary nodes and the value of their displacements and tractions (real and imaginary part) along x_1 and x_2, as calculated through a system of boundary integral equations;
- the coordinates x_1 and x_2 of internal nodes and the value of displacement (real and imaginary part) along x_1 and x_2 (*u1r, u2r, u1i, u2i*);
- the coordinates x_1 and x_2 of internal nodes and the gradient of displacement (real and imaginary part) along x_1 and x_2 (*u1,1r, u1,2r, u2,1r, u2,2r, u1,1i, u1,2i, u2,1i, u2,2i*);
- the coordinates x_1 and x_2 of internal nodes and the value of tractions (real and imaginary part) along x_1 and x_2 (*t11r, t12r, t22r, t21r, t11i, t12i, t22i, t21i*) and of pressure (*Pressr, Pressi*);
- the coordinates x_1 and x_2 of internal nodes, the value of principal stresses and the Von Mises stress (real and imaginary part) along x_1 and x_2 (*s11r, s12r, s22r, s21r, VonMisesr, s11i, s12i, s22i, s21i, VonMisesi*).

Note that internal nodes are automatically generated as in Fig. 3.

Bibliography

[1] D. Bigoni, D. Capuani, P. Bonetti, S. Colli. A novel boundary element approach to time-harmonic dynamics of incremental non-linear elasticity: the role of pre-stress on structural vibrations and dynamic shear banding. *Comp. Meth. Appl. Mech. Eng.*, (**2007**), in press.